

$$\boxed{\varphi \vee \psi \vdash \psi \vee \varphi}$$

$$\frac{\varphi \vee \psi \vdash \varphi \vee \psi \quad \frac{\varphi \vee \psi, \varphi \vdash \varphi \quad \varphi \vee \psi, \psi \vdash \varphi \vee \psi}{\varphi \vee \psi, \varphi \vdash \varphi \vee \psi} \quad \frac{\varphi \vee \psi, \psi \vdash \varphi \vee \psi \quad \varphi \vee \psi, \psi \vdash \psi}{\varphi \vee \psi, \psi \vdash \varphi \vee \psi}}{\varphi \vee \psi \vdash \psi \vee \varphi} \quad (ve)$$

$$\frac{\varphi \vee \psi \vdash \varphi \quad \varphi \vee \psi \vdash \psi}{\varphi \vee \psi \vdash \psi \vee \varphi} \quad (vi)$$

$$\frac{\frac{\varphi \vee \psi \vdash \varphi \quad \varphi \vee \psi \vdash \psi}{\varphi \vee \psi \vdash \psi \vee \varphi} \quad \varphi \vee \psi \vdash \varphi \vee \psi}{\varphi \vee \psi \vdash \psi \vee \varphi} \quad (cvt)$$

$$\mathcal{T} = \{ \varphi_1 \vee \varphi_2, \varphi_1 \rightarrow \varphi_2, \varphi_2 \rightarrow \varphi_1, \varphi_2 \}$$

$$\frac{\mathcal{T} \vdash \varphi_2 \quad \mathcal{T}, \varphi_2 \vdash \varphi_1}{\mathcal{T} \vdash \varphi_1} \quad (?)$$

$$\boxed{(\neg A) \vee B \vdash A \rightarrow B}$$

$$\frac{\frac{\frac{\frac{\neg A \vee B, A \vdash \neg A \quad \neg A \vee B, A \vdash A}{\neg A \vee B, A \vdash \perp} \quad (\neg \vee)}{\neg A \vee B, A \vdash \perp} \quad (\neg \vee)}{\neg A \vee B, A \vdash B} \quad (\neg \vee)}{\neg A \vee B, A \vdash B} \quad (\neg \vee)}{\neg A \vee B \vdash A \rightarrow B} \quad (\rightarrow \vee)$$

$$\varphi, \neg \varphi \vdash \neg \varphi$$

$$\frac{\frac{\varphi, \neg \varphi \vdash \neg \varphi \quad \varphi, \neg \varphi \vdash \varphi}{\varphi, \neg \varphi \vdash \perp} \quad (\neg \vee)}{\varphi, \neg \varphi \vdash \neg \varphi} \quad (vi)$$