Ex 51: $|l_1 \circ l_2| = |l_1| + |l_2|, \forall l_1, l_2.$

Prova: Indução sobre a estrutura da lista l_1.

① $l_1 = nil$: $|l_1 \circ l_2| = |nil \circ l_2| \overset{def.}{=} |l_2| = 0 + |l_2| =$

$= |nil| + |l_2| = |l_1| + |l_2|.$

② $l_1 = h :: l_1'$: $|l_1 \circ l_2| = |\underline{(h :: l_1') \circ l_2}| \overset{def.}{=} |h :: (l_1' \circ l_2)| \overset{def.}{=}$

$1 + \underbrace{|l_1' \circ l_2| \overset{(h.i.)}{=} 1 + |l_1'| + |l_2|}_{(h.i.)} \overset{def.}{=} |(h :: l_1')| + |l_2| = |l_1| + |l_2|$ $\boxed{}$

Ex. 52: $l \circ nil = l, \forall l.$

Prova: Indução na estrutura de l:

① $l = nil$: $l \circ nil = nil \circ nil \overset{def.}{=} nil = l.$ ✓

② $l = h :: l'$: $l \circ nil = (h :: l') \circ nil =$

$h :: (l' \circ nil) \overset{(h.i.)}{=} h :: l' = l.$ $\boxed{}$

Ex 53: $(l_1 \circ l_2) \circ l_3 = l_1 \circ (l_2 \circ l_3), \forall l_1, l_2, l_3.$

Prova: Indução na estrutura de l_1:

① $l_1 = nil$: $(l_1 \circ l_2) \circ l_3 = (nil \circ l_2) \circ l_3 \overset{def.}{=}$

$l_2 \circ l_3 \overset{def}{=} nil \circ (l_2 \circ l_3) = l_1 \circ (l_2 \circ l_3).$

② $l_1 = h :: l_1'$: $(l_1 \circ l_2) \circ l_3 = (\underline{(h :: l_1') \circ l_2}) \circ l_3 \overset{def.}{=}$

$\underline{(h :: (l_1' \circ l_2)) \circ l_3} \overset{def.}{=} h :: (\underline{(l_1' \circ l_2) \circ l_3}) \overset{h.i.}{=}$

$h :: (l_1' \circ (l_2 \circ l_3)) \overset{def.}{=} (h :: l_1') \circ (l_2 \circ l_3) = l_1 \circ (l_2 \circ l_3).$ $\boxed{}$

BI $\underbrace{\forall \ell_2,\ |nil \circ \ell_2| = |nil| + |\ell_2|}_{}$ $P(nil)$ ✓

PI $\forall \ell_1'\, h,\ P(\ell_1') \to P(h :: \ell_1')$ ✓

$\forall \ell_1'\, h,\ (\forall \ell_2,\ |\ell_1' \circ \ell_2| = |\ell_1'| + |\ell_2|) \to (\forall \ell_2,\ |(h :: \ell_1') \circ \ell_2| =$

$|(h :: \ell_1')| + |\ell_2|$