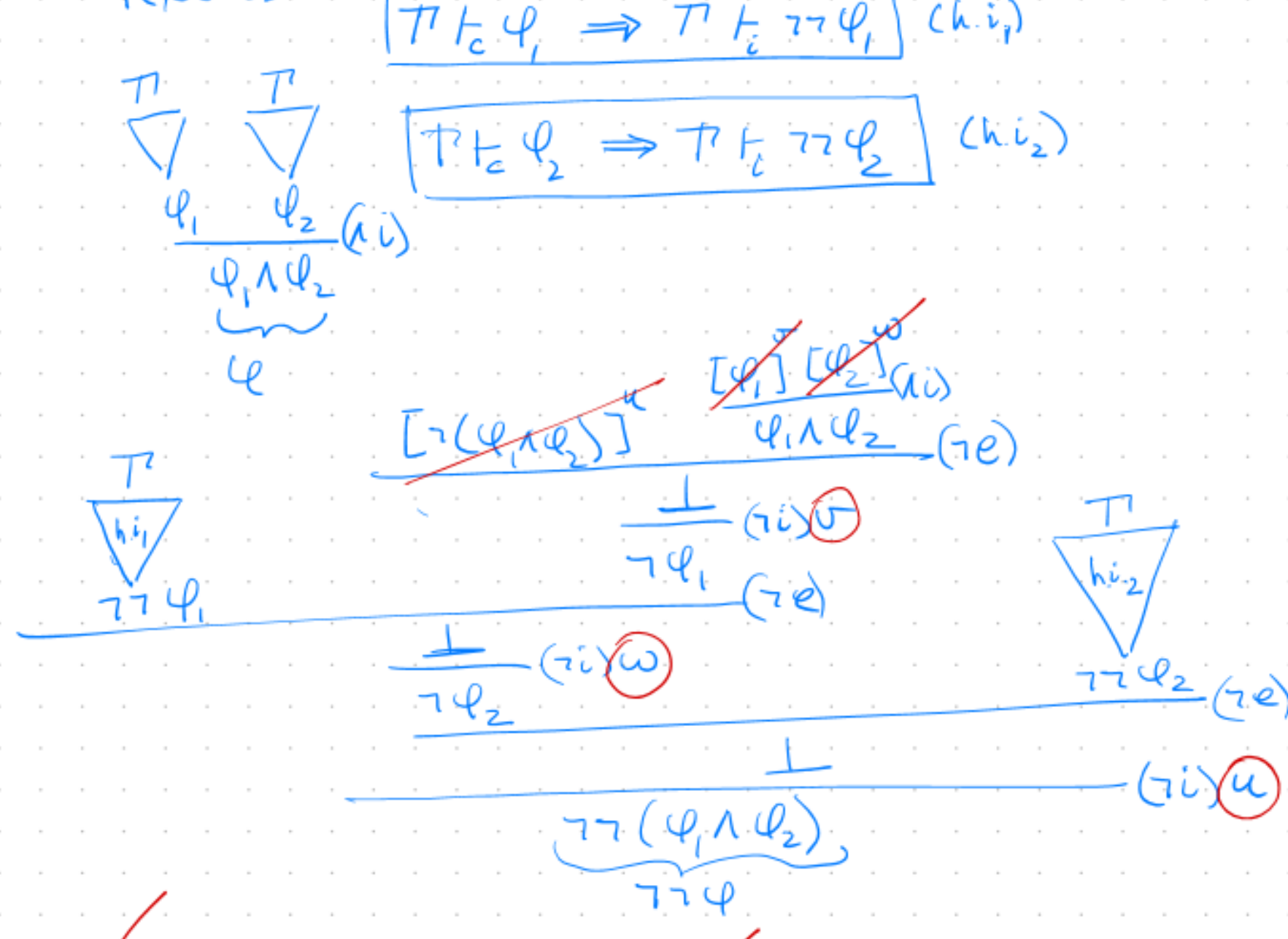


Na aula de hoje, iniciamos a prova do Teorema de Glivenko. O rascunho das anotações da aula estão a seguir:

Teorema de Glivenko: $\vdash \vdash \varphi \Rightarrow \vdash \vdash \neg \neg \varphi$

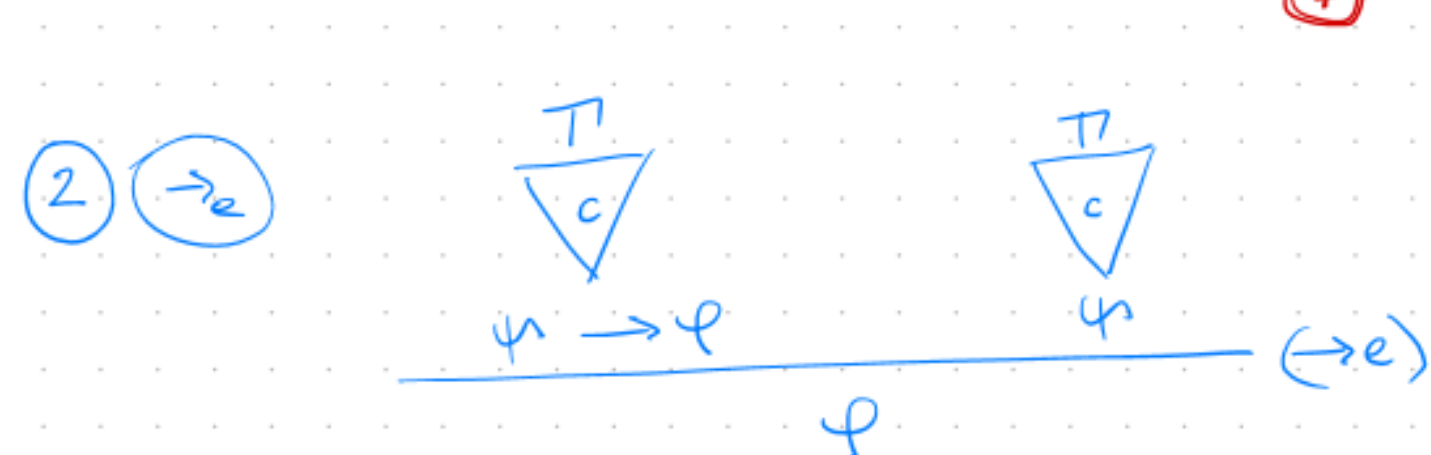
Prova: Indução em $\vdash \vdash \varphi$. Considere a última regra que gera a derivação $\vdash \vdash \varphi$:

① (h.i) Se a última regra de $\vdash \vdash \varphi$ é (h.i), temos:



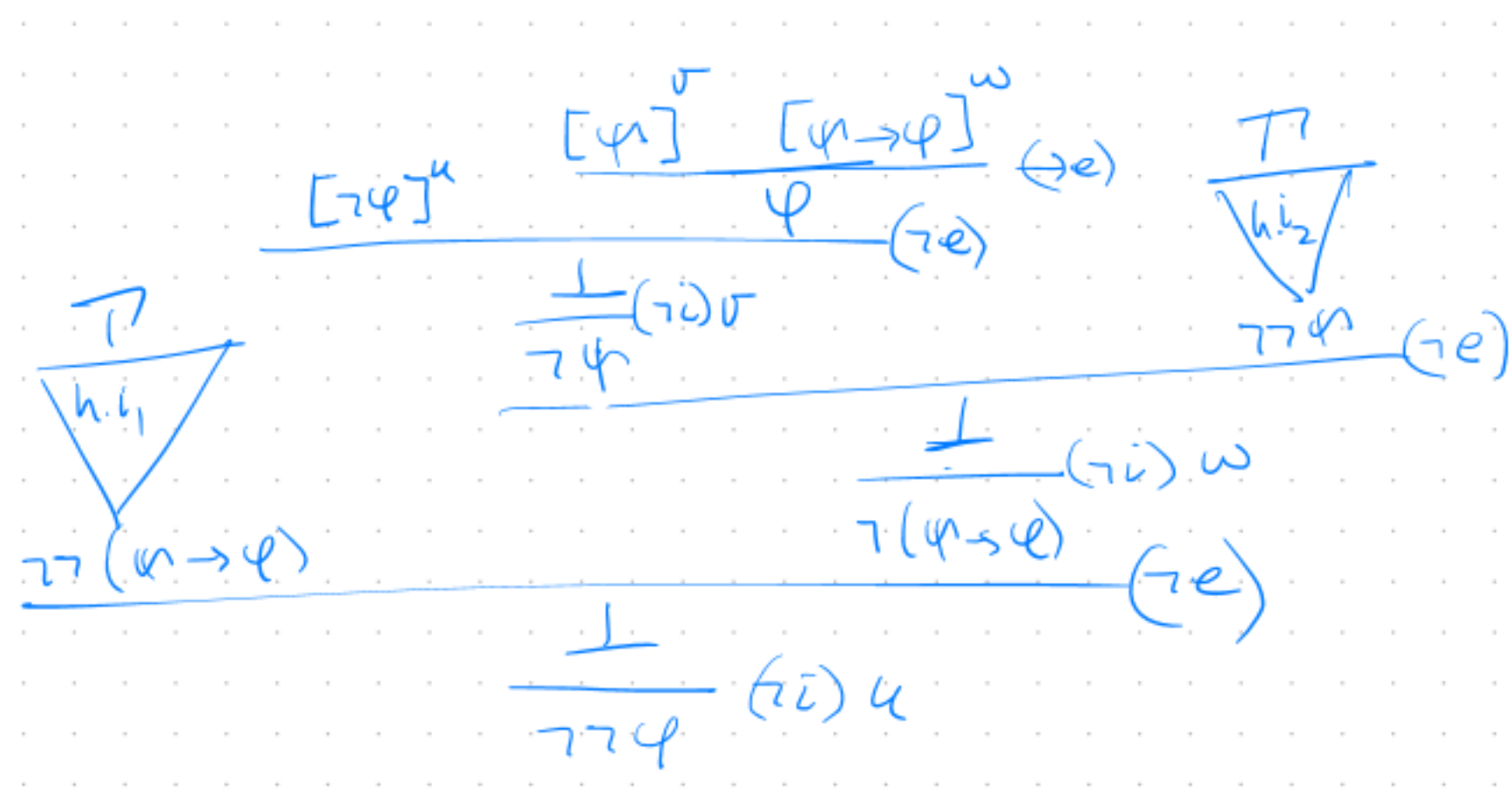
LPC: $\neg i, \neg e, \vee i, \vee e, \rightarrow i, \rightarrow e$,

LP
PBC
$\neg e$
LTE



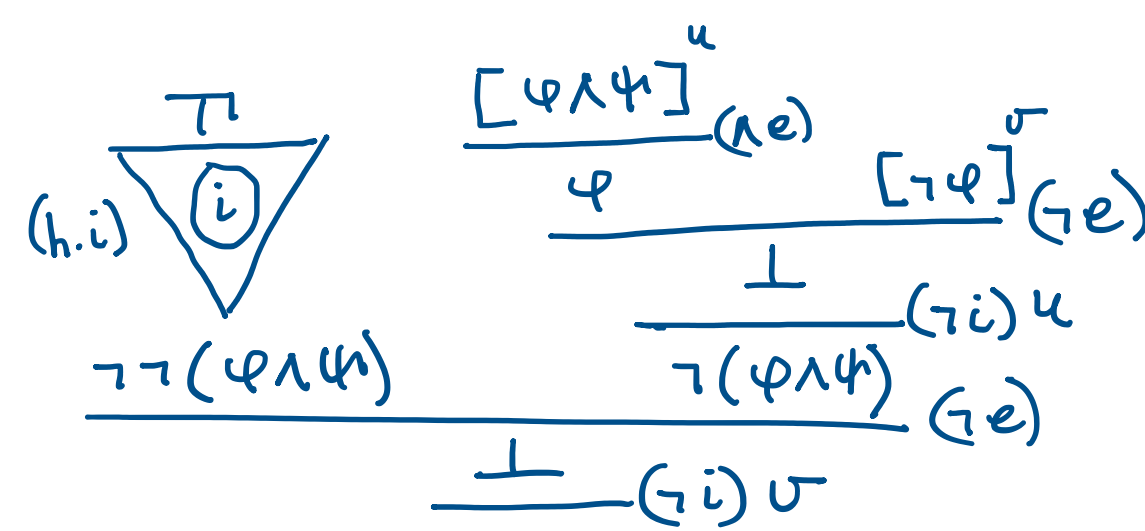
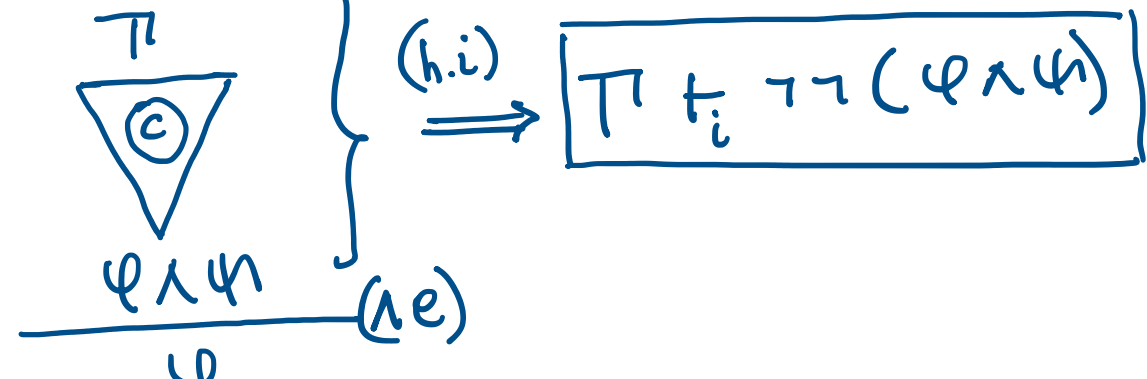
$\vdash \vdash \varphi \rightarrow \varphi \Rightarrow \vdash \vdash \neg \neg (\varphi \rightarrow \varphi)$ (h.i)

$\vdash \vdash \varphi \Rightarrow \vdash \vdash \neg \neg \varphi$ (h.i)

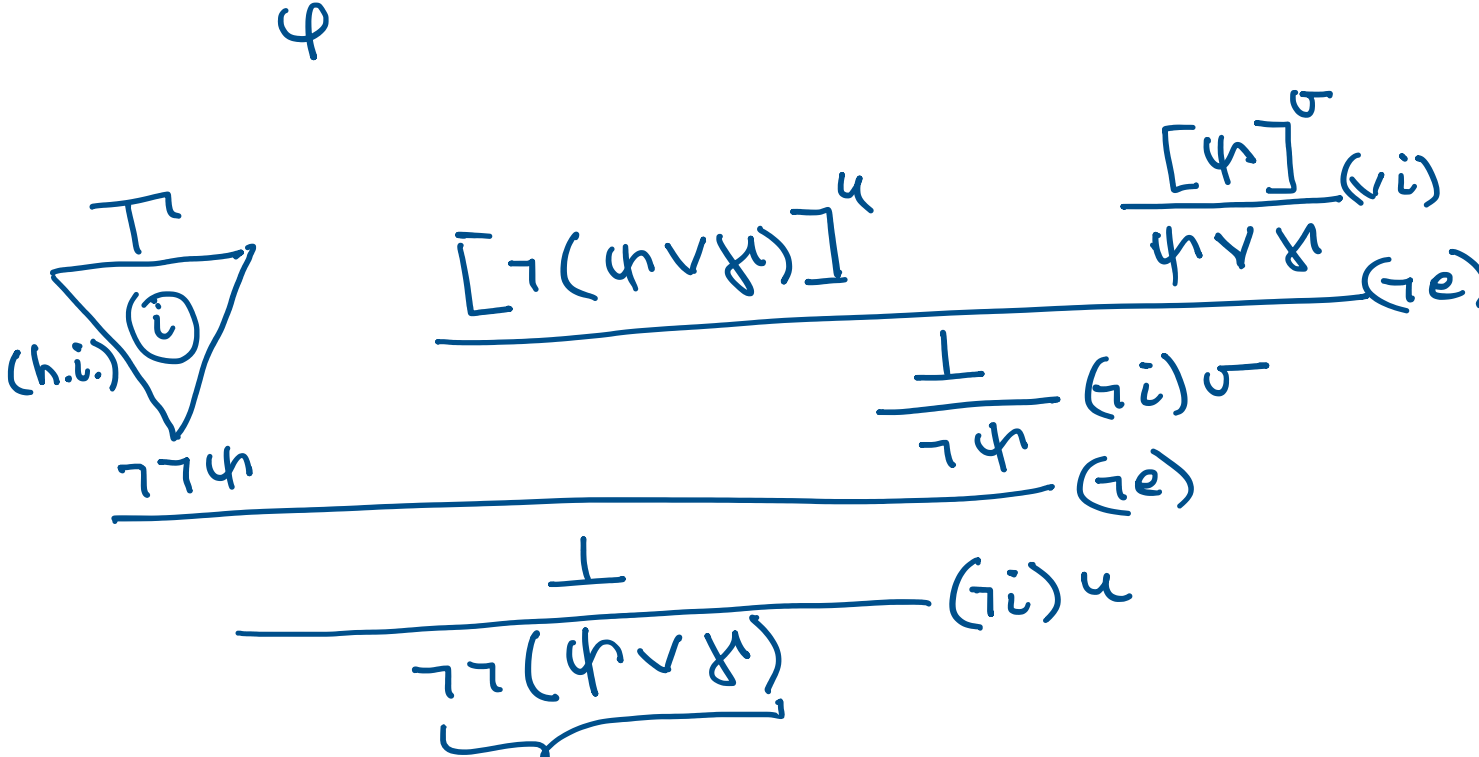
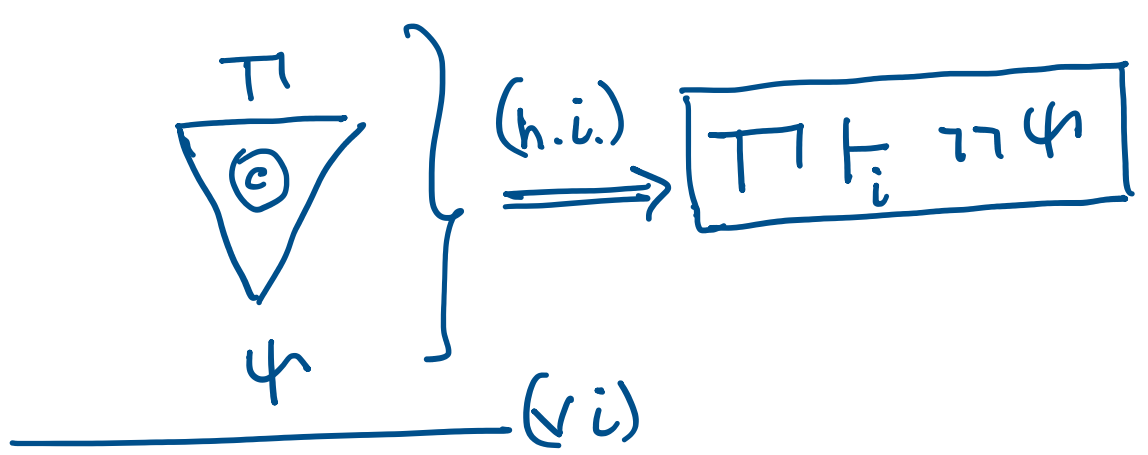


Continue a prova fazendo pelo menos um dos casos que não foram feitos acima.

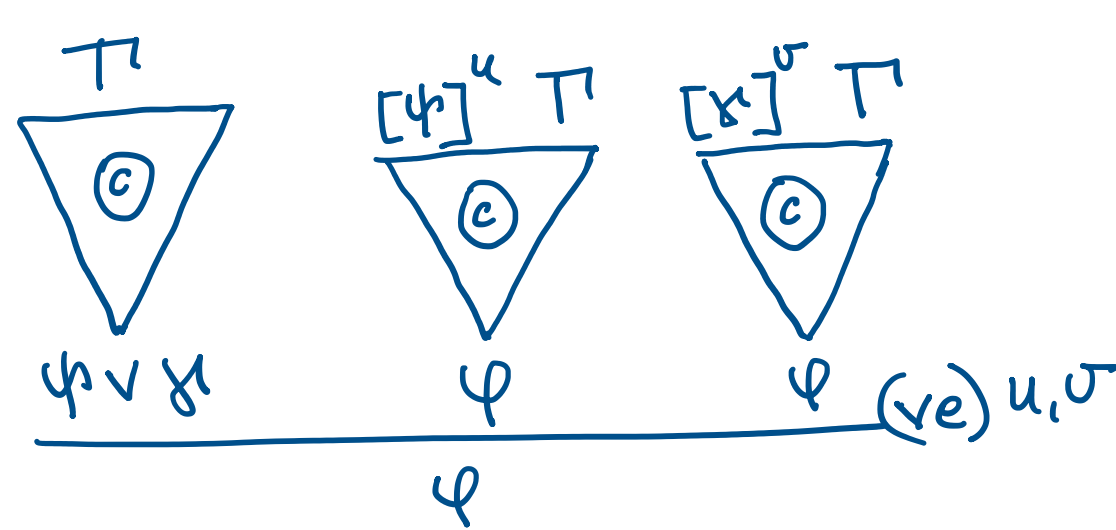
② (h.e)



③ (v.i)



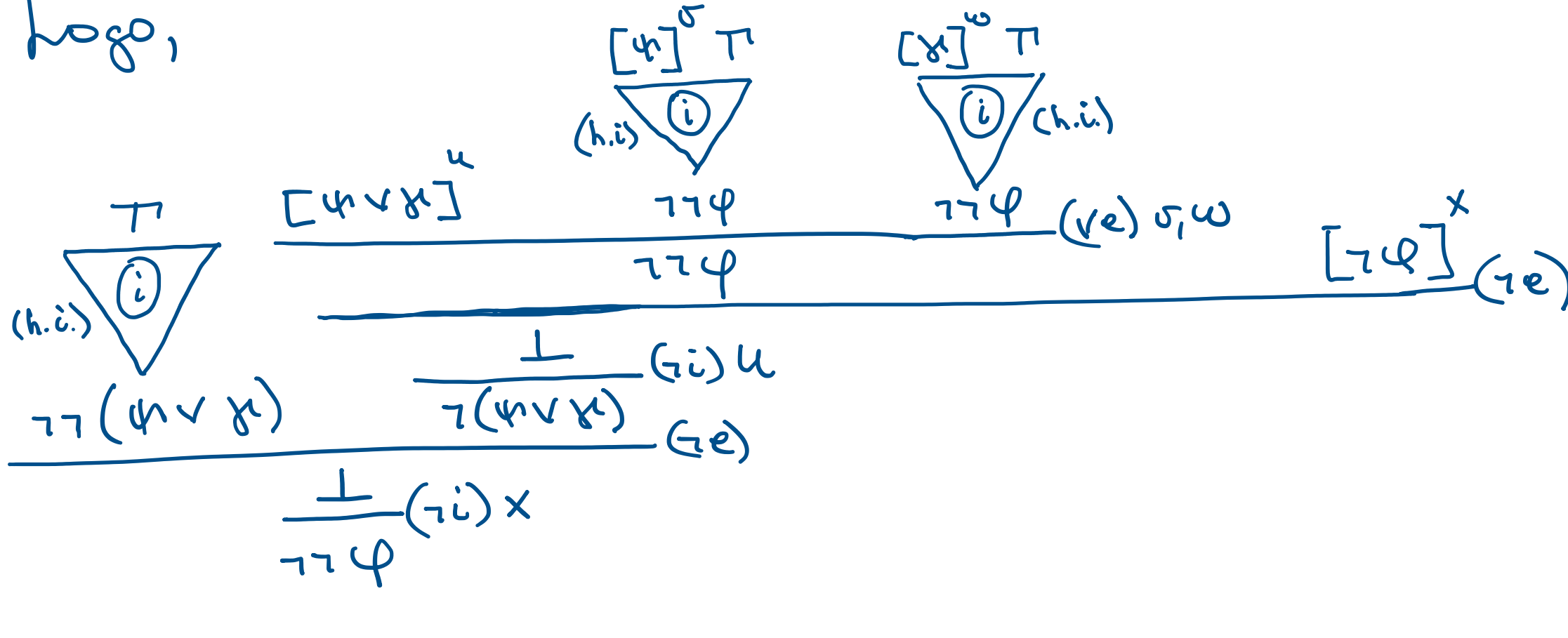
④ (v.e)



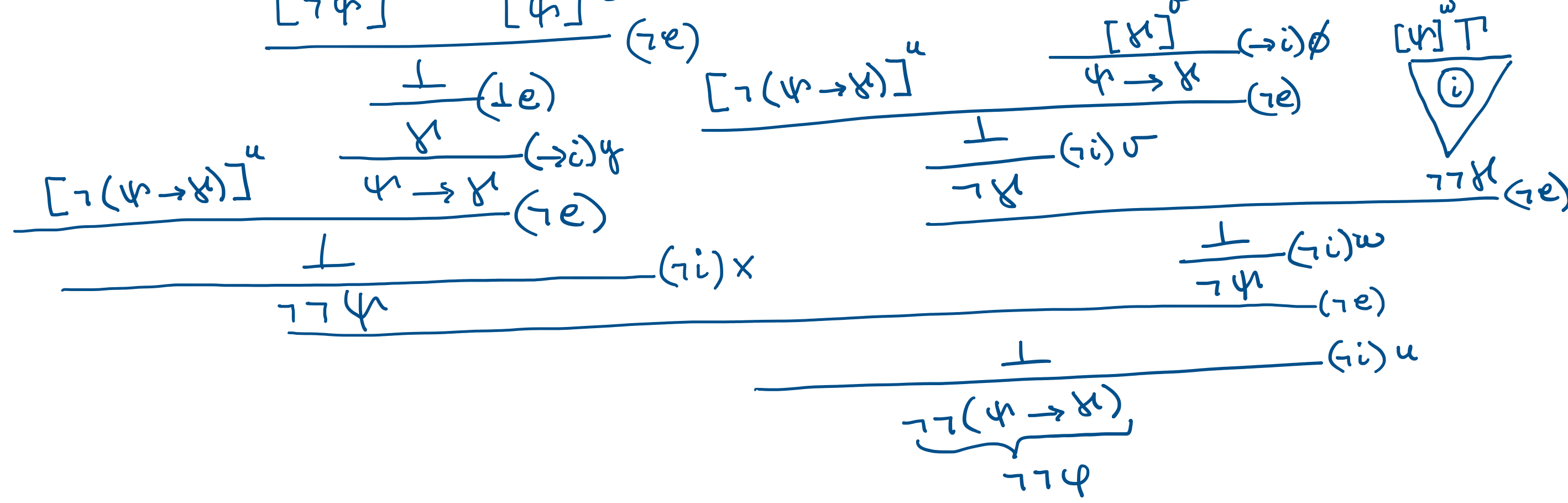
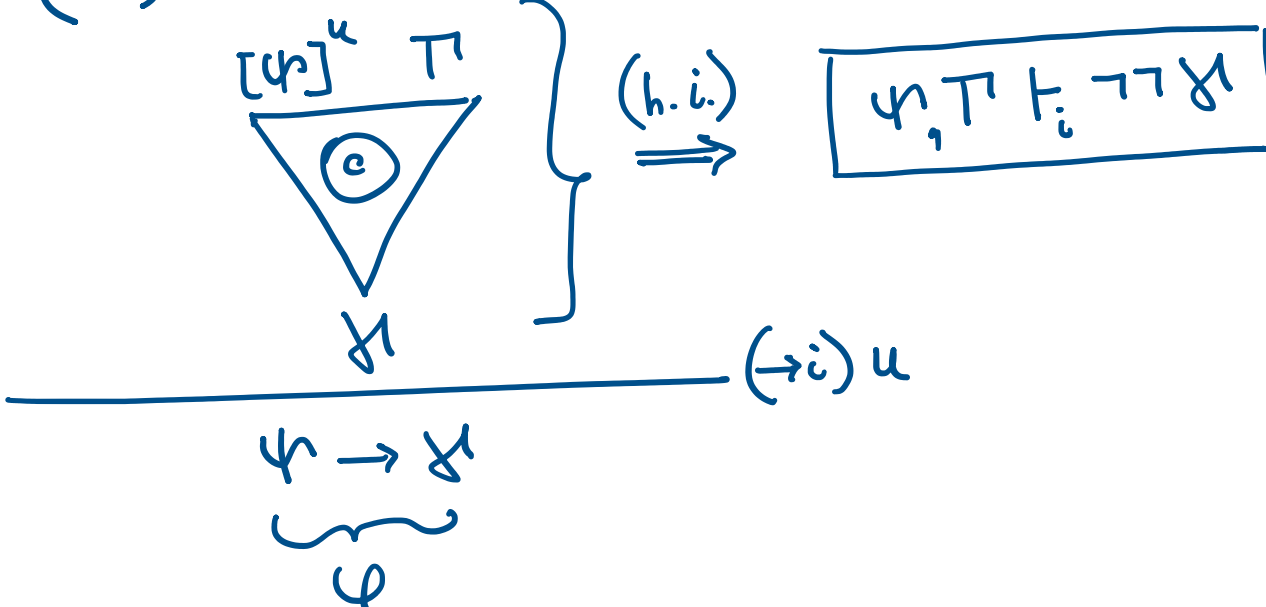
Temos, por hipótese de indução:

$\vdash \vdash \neg \neg (\varphi \vee \psi)$
$\varphi, \vdash \vdash \neg \neg \varphi$
$\psi, \vdash \vdash \neg \neg \psi$

Logo,



⑤ ($\rightarrow i$)



⑥ (PBC)

