Exercício 16:

$$\dfrac{\dfrac{\dfrac{}{(\neg\varphi)\vee(\neg\gamma)\vdash(\neg\varphi)\vee(\neg\gamma)}\,(Ax) \qquad \dfrac{\dfrac{}{\varphi\wedge\gamma,\neg\varphi\vdash\neg\varphi}\,(Ax) \quad \dfrac{\dfrac{}{\varphi\wedge\gamma,\neg\varphi\vdash\varphi\wedge\gamma}\,(Ax)}{\varphi\wedge\gamma,\neg\varphi\vdash\varphi}\,(\wedge e)}{\varphi\wedge\gamma,\neg\varphi\vdash\bot}\,(\neg e) \qquad \dfrac{\dfrac{}{\varphi\wedge\gamma,\neg\gamma\vdash\neg\gamma}\,(Ax) \quad \dfrac{\dfrac{}{\varphi\wedge\gamma,\neg\gamma\vdash\varphi\wedge\gamma}\,(Ax)}{\varphi\wedge\gamma,\neg\gamma\vdash\gamma}\,(\wedge e)}{\varphi\wedge\gamma,\neg\gamma\vdash\bot}\,(\neg e)}{(\neg\varphi)\vee(\neg\gamma),\varphi\wedge\gamma\vdash\bot}\,(\vee e)}{(\neg\varphi)\vee(\neg\gamma)\vdash\neg(\varphi\wedge\gamma)}\,(\neg i)$$

$\Longrightarrow$ Outras soluções são possíveis! Veja abaixo:

$$\dfrac{\dfrac{\dfrac{}{(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma),(\neg\gamma),\varphi\vdash\neg\gamma}\,(Ax) \qquad \dfrac{\dfrac{}{(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma),(\neg\gamma),\varphi\vdash(\varphi\wedge\gamma}\,(Ax)}{(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma),(\neg\gamma),\varphi\vdash\gamma}\,(\wedge e)}{(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma),(\neg\gamma),\varphi\vdash\bot}\,(\neg e)}{**\,(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma),(\neg\gamma)\vdash\neg\varphi}\,(\neg\wedge)$$

$$\dfrac{}{*\,(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma),(\neg\varphi)\vdash(\neg\varphi)}\,(Ax$$

$$\dfrac{\dfrac{\dfrac{}{(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma)\vdash(\neg\varphi)\vee(\neg\gamma)}\,(Ax) \quad * \quad **}{(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma)\vdash\neg\varphi}\,(\vee e) \qquad \dfrac{\dfrac{}{(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma)\vdash(\varphi\wedge\gamma)}\,(Ax)}{(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma)\vdash\varphi}\,(\wedge e)}{(\neg\varphi)\vee(\neg\gamma),(\varphi\wedge\gamma)\vdash\bot}\,(\neg e)}{(\neg\varphi)\vee(\neg\gamma)\vdash\neg(\varphi\wedge\gamma)}\,(\neg i)$$

$$\dfrac{\dfrac{\dfrac{}{(\neg\varphi)\vee(\neg\gamma)\vdash(\neg\varphi)\vee(\neg\gamma)}\,(Ax) \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\neg\varphi,\varphi\wedge\gamma\vdash\neg\varphi}\,(Ax) \quad \dfrac{\dfrac{}{\neg\varphi,\varphi\wedge\gamma\vdash\varphi\wedge\gamma}\,(Ax)}{\neg\varphi,\varphi\wedge\gamma\vdash\varphi}\,(\wedge e)}{\neg\varphi,\varphi\wedge\gamma\vdash\bot}\,(\neg e)}{\neg\varphi\vdash\neg(\varphi\wedge\gamma)}\,(\neg i) \qquad \dfrac{\dfrac{\dfrac{}{\neg\gamma,\varphi\wedge\gamma\vdash\neg\gamma}\,(Ax) \quad \dfrac{\dfrac{}{\neg\gamma,\varphi\wedge\gamma\vdash\varphi\wedge\gamma}\,(Ax)}{\neg\gamma,\varphi\wedge\gamma\vdash\gamma}\,(\wedge e)}{\neg\gamma,\varphi\wedge\gamma\vdash\bot}\,(\neg e)}{\neg\gamma\vdash\neg(\varphi\wedge\gamma)}}{(\neg\varphi)\vee(\neg\gamma)\vdash\neg(\varphi\wedge\gamma)}\,(\vee e)$$