

Atividade 3:  $\vdash_i \neg \neg ((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi$

Solução 1

$$\begin{array}{c}
 \frac{[\varphi]^u \quad [\neg\varphi]^v}{\perp} (\perp e) \\
 \frac{\perp}{\varphi} (\perp e) \\
 \frac{\varphi}{\varphi \rightarrow \varphi} (\rightarrow i) u \\
 \frac{[\neg\varphi]^v \quad \varphi}{\perp} (\perp e) \\
 \frac{\perp}{\neg((\varphi \rightarrow \psi) \rightarrow \varphi)} (\neg i) v \\
 \frac{\neg((\varphi \rightarrow \psi) \rightarrow \varphi)}{\neg \neg((\varphi \rightarrow \psi) \rightarrow \varphi)} (\neg e) \\
 \frac{\neg \neg((\varphi \rightarrow \psi) \rightarrow \varphi)}{\neg \neg \varphi} (\rightarrow_i) \exists \\
 \frac{\neg \neg \varphi}{\neg \neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \neg \neg \varphi} (\rightarrow e) \\
 \frac{\neg \neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \neg \neg \varphi}{\neg \neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi} (*) \quad \boxed{\text{Ex. 37}}
 \end{array}$$

onde

$$\neg \neg A \rightarrow \neg \neg B \vdash_i \neg \neg (A \rightarrow B) \quad (*)$$

$$\begin{array}{c}
 \frac{[A]^u \quad [\neg A]^w}{\perp} (\perp e) \\
 \frac{\perp}{B} (\perp e) \\
 \frac{B}{A \rightarrow B} (\rightarrow i) v \\
 \frac{A \rightarrow B}{\neg \neg(A \rightarrow B)} (\neg e) \\
 \frac{\perp}{\neg \neg A} (\neg i) w \\
 \frac{\neg \neg A \quad \neg \neg(A \rightarrow B)}{\neg \neg B} (\rightarrow e) \\
 \frac{[B]^z}{A \rightarrow B} (\rightarrow_i) \phi \\
 \frac{A \rightarrow B \quad [\neg(A \rightarrow B)]^u}{\perp} (\perp e) \\
 \frac{\perp}{\neg B} (\neg i) \exists \\
 \frac{\neg B}{\neg \neg(A \rightarrow B)} (\neg e)
 \end{array}$$

Solução 2:

$$\begin{array}{c}
 \frac{[\neg((\varphi \rightarrow \psi) \rightarrow \varphi)]^u \quad \frac{[\varphi]^v}{((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi} (\rightarrow_i) \phi}{\perp} (\perp e) \\
 \frac{\perp}{\varphi} (\perp e) \\
 \frac{\varphi}{(\varphi \rightarrow \psi) \rightarrow \varphi} (\rightarrow i) v \\
 \frac{(\varphi \rightarrow \psi) \rightarrow \varphi}{\varphi} (\rightarrow e) \\
 \frac{\varphi}{((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi} (\rightarrow_i) w \\
 \frac{((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi \quad [\neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi]^u}{\perp} (\perp e) \\
 \frac{\perp}{\neg \neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi} (\neg i)
 \end{array}$$