Atividade 7:

Quando $\varphi = \varphi_1 \wedge \varphi_2$, temos $\varphi^N = (\varphi_1 \wedge \varphi_2)^N = \varphi_1^N \wedge \varphi_2^N$.

Logo, temos que provar o sequente $\neg\neg(\varphi_1^N \wedge \varphi_2) \vdash_i \varphi_1^N \wedge \varphi_2^N$.

Temos, por hipótese de indução, que $\neg\neg\varphi_1^N \vdash_i \varphi_1^N$ e $\neg\neg\varphi_2^N \vdash_i \varphi_2^N$.

Logo,

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg\varphi_1^N]^u \qquad \dfrac{[\varphi_1^N \wedge \varphi_2^N]^v}{\varphi_1^N}\,(\wedge e)}{\bot}\,(\neg e)}{\neg(\varphi_1^N \wedge \varphi_2^N)}\,(\neg i)\,v \qquad \neg\neg(\varphi_1^N \wedge \varphi_2^N)}{\bot}\,(\neg e)}{\neg\neg\varphi_1^N}\,(\neg i)\,u}{\varphi_1^N}\,(h.i)
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg\varphi_2^N]^w \qquad \dfrac{[\varphi_1^N \wedge \varphi_2^N]^z}{\varphi_2^N}\,(\wedge e)}{\bot}\,(\neg e)}{\neg(\varphi_1^N \wedge \varphi_2^N)}\,(\neg i)\,z \qquad \neg\neg(\varphi_1^N \wedge \varphi_2^N)}{\bot}\,(\neg e)}{\neg\neg\varphi_2^N}\,(\neg i)\,w}{\varphi_2^N}\,(h.i)
}{\varphi_1^N \wedge \varphi_2^N}\,(\wedge i)
$$