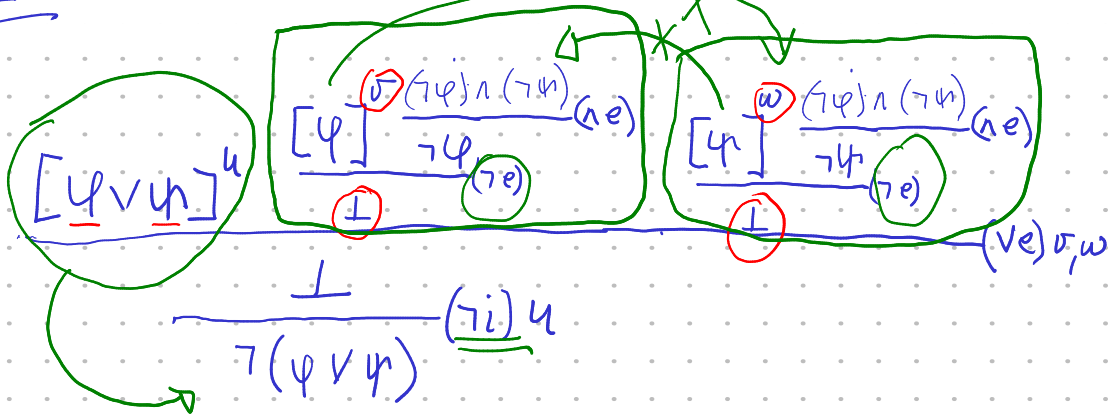


**Exercício 19.** Sejam  $\varphi$  e  $\psi$  fórmulas da lógica proposicional. Construa uma prova para o sequente  $(\neg\varphi) \wedge (\neg\psi) \vdash \neg(\varphi \vee \psi)$  na lógica proposicional minimal.



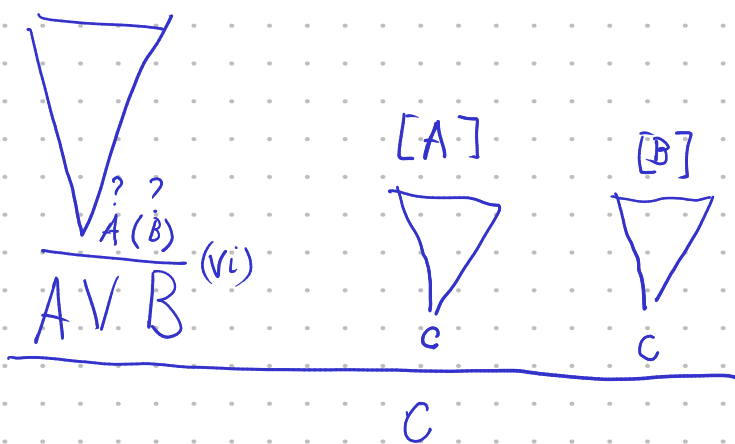
$$\frac{\top \vdash A}{\top \vdash \neg\neg A} (\neg\neg i)$$

$$\frac{A, [\neg A]^u}{\perp} (\neg e)$$

$$\frac{\perp}{\neg\neg A} (\neg i)$$

$$\frac{\frac{[\neg(\varphi) \wedge (\neg\psi)]^u}{\perp} (\neg e)}{\neg(\neg\varphi) \wedge (\neg\psi)} (\neg i)$$

$$\frac{(\varphi \vee \psi) \rightarrow (\neg\varphi) \wedge (\neg\psi) \quad \neg(\neg\varphi) \wedge (\neg\psi)}{\neg(\varphi \vee \psi)}$$



$$\vdash_m \neg\neg(A \vee \neg A)$$

$$\boxed{\neg\neg(\neg A) \vdash_m (\neg A)}$$

$$[\neg A]^u$$

$$\vdots$$

$$\frac{\perp}{A} (RAA)_u$$

$$\boxed{(\neg A) \rightarrow \perp \vdash A}$$

$$\vdash A \vee (\neg A)$$

$$\frac{\neg\neg A}{A} (\neg\neg e)$$

$$\boxed{\neg\neg A \vdash A}$$

LPI

$$\frac{\frac{[A]^u}{B} (\rightarrow_i) u}{A \rightarrow B} (\rightarrow_i) u$$

$$\boxed{B \vdash A \rightarrow B}$$

$$\boxed{\frac{B}{A \rightarrow B} (\rightarrow_i) \phi}$$

$$\boxed{\frac{[A]^u \quad B}{A \rightarrow B} (\rightarrow_i) u}$$

$$\frac{\frac{\frac{[A]^u \quad B}{A \wedge B} (\wedge_i)}{B} (\wedge_e)}{A \rightarrow B} (\rightarrow_i) u$$

$$\frac{B, A \vdash B (\text{Ax})}{B \vdash A \rightarrow B} (\rightarrow_i)$$

LPI = LPM + (Le)

$$\boxed{\frac{\perp}{\phi} (\perp_e)}$$

regra de explosão.

Existem irracionais  $a$  e  $b$  tais que  $a^b$  é racional.

$$\left. \begin{array}{l} \left( \sqrt{2}^{\sqrt{2}} \in \mathbb{Q} \right) \vee \left( \sqrt{2}^{\sqrt{2}} \in \mathbb{R} \setminus \mathbb{Q} \right) \\ \left. \begin{array}{l} a = b = \sqrt{2} \\ \neg (\sqrt{2}^{\sqrt{2}} \in \mathbb{Q}) \end{array} \right\} \begin{array}{l} a = \sqrt{2}^{\sqrt{2}} \\ b = \sqrt{2} \\ a^b = 2 \end{array} \end{array} \right.$$

$$A \vdash \neg A \vdash_i B$$

$$\left. \begin{array}{l} \frac{A \quad \neg A}{\perp} (\neg e) \\ \frac{\perp}{B} (\perp e) \end{array} \right\} \frac{\psi \quad \neg \psi}{\psi} (\neg e)'$$

$$A, \neg A \vdash_m \neg B$$

$$\begin{array}{l} \frac{[B]^u \quad A}{B \wedge A} (\wedge i) \\ \frac{B \wedge A}{A} (\wedge e) \\ \frac{A \quad \neg A}{\perp} (\neg e) \\ \frac{\perp}{\neg B} (\neg i) u \end{array}$$