



$$\begin{array}{l}
 \frac{\frac{\frac{(Ax)}{\neg\psi \vdash \neg\psi} \quad \frac{(Ax)}{\psi \vdash \psi} (\neg e)}{\neg\psi, \psi \vdash \perp} (Le)}{\neg\psi, \psi \vdash \psi} (*) (ve)}{\frac{(Ax)}{(\neg\psi) \vee \psi \vdash (\neg\psi) \vee \psi} \quad \frac{(Ax)}{(\neg\psi) \vee \psi, \psi \vdash \psi}}{(\neg\psi) \vee \psi \vdash \psi \rightarrow \psi} (\rightarrow i)}
 \end{array}$$

$$\frac{\frac{\frac{[\psi]^w \quad [\psi]^u}{\perp} (\neg e)}{\frac{\perp}{\psi} (Le)} \quad \frac{[\psi]^v}{(\neg\psi) \vee \psi} (ve)_u}{\psi} (\rightarrow i)_w$$

$$\frac{\frac{(Ax) \text{ se } A \in \mathcal{T}}{\mathcal{T} \vdash A} \quad \frac{(Ax)}{A \vdash A}}{\mathcal{T} \vdash A}$$

$$\frac{\frac{\frac{[A]^u \quad B}{A \wedge B} (\wedge i)}{A \wedge B} (\wedge e)}{B} (\rightarrow i)_u}{A \rightarrow B}$$

$$\frac{\frac{\frac{(Ax)}{A \vdash A} \quad \frac{(Ax)}{B \vdash B} (\wedge i)}{B, A \vdash A \wedge B} (\wedge e)}{B, A \vdash B} (\wedge e)}{B \vdash A \rightarrow B} (\rightarrow i)$$

$$\frac{\mathcal{T}_1 \vdash A \quad \mathcal{T}_2 \vdash B}{\mathcal{T}_1, \mathcal{T}_2 \vdash A \wedge B} (\wedge i)$$

$$\frac{\mathcal{T} \vdash A_1 \vee A_2 (vi)}{\mathcal{T} \vdash A_1 \vee A_2}$$

$$\begin{array}{c}
 [A] \\
 \vdots \\
 A \vee (B \vee C) \\
 \hline
 \end{array}
 \quad
 \begin{array}{c}
 [B \vee C] \\
 \vdots \\
 \end{array}
 \quad
 (ve)$$

$$A \vee (B \vee C) \vdash (A \vee B) \vee C$$

Exercício 18. Sejam φ e ψ fórmulas da lógica proposicional. Construa uma prova para o sequente $\neg(\varphi \vee \psi) \vdash (\neg\varphi) \wedge (\neg\psi)$ na lógica proposicional minimal.

$$\begin{array}{c}
 [\varphi]^u \\
 \vdots \\
 \hline
 \neg\varphi \quad (\neg i)^u
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{c}
 \frac{[\varphi]^u}{\varphi \vee \psi} \quad (\vee i) \\
 \neg(\varphi \vee \psi) \quad \perp \\
 \hline
 \neg\varphi \quad (\neg i)^u
 \end{array}
 \quad
 \begin{array}{c}
 \frac{[\psi]^v}{\varphi \vee \psi} \quad (\vee i) \\
 \neg(\varphi \vee \psi) \quad \perp \\
 \hline
 \neg\psi \quad (\neg i)^v
 \end{array} \\
 \frac{\neg\varphi \quad \neg\psi}{(\neg\varphi) \wedge (\neg\psi)} \quad (\wedge i)
 \end{array}$$