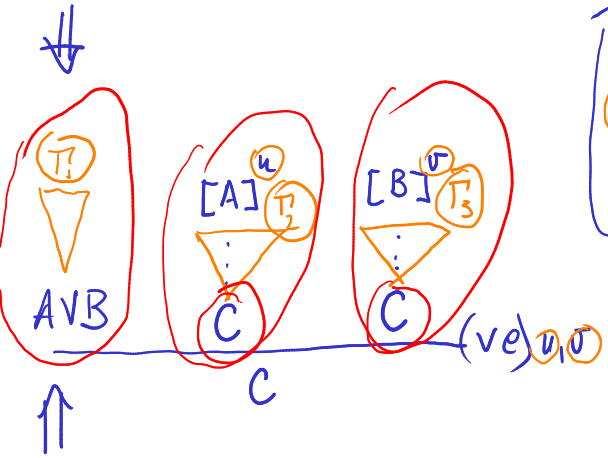


$$\frac{\Gamma_1 \vdash A \vee B \qquad \Gamma_2, A \vdash C \qquad \Gamma_3, B \vdash C}{\Gamma_1, \Gamma_2, \Gamma_3 \vdash C}\ (\vee e)$$

$$\frac{}{\gamma \vdash \gamma}\ (Ax)$$

$$\frac{\dfrac{\dfrac{\dfrac{}{(\neg\varphi)\vee\gamma \vdash (\neg\varphi)\vee\gamma}(Ax) \qquad \dfrac{}{\varphi \vdash \varphi}(Ax)}{\neg\varphi, \varphi \vdash \bot}(\neg e) \qquad \dfrac{}{\gamma \vdash \gamma}(Ax)}{(\neg\varphi)\vee\gamma, \varphi \vdash \gamma}(\bot e)}{(\neg\varphi)\vee\gamma \vdash \varphi \to \gamma}(\vee e)\ (\to i)$$

$$\frac{\dfrac{\dfrac{[\varphi]^{\omega} \quad [\neg\varphi]^{u}}{\bot}(\neg e) \qquad [\gamma]^{v}}{\gamma}(\bot e)}{\varphi \to \gamma}(\vee e)^{v}_{u}\ (\to i)\,\omega$$

$$\frac{\Gamma \vdash A \quad \text{ile } A \in \Gamma}{\Gamma \vdash A}\ (Ax)$$

$$\frac{}{A \vdash A}\ (Ax)$$

$$B \vdash A \to B$$

$$\left\{\begin{array}{l}\dfrac{\dfrac{[A]^{u} \quad B}{A \wedge B}(\wedge i)}{B}(\wedge e)\end{array}\right\}$$

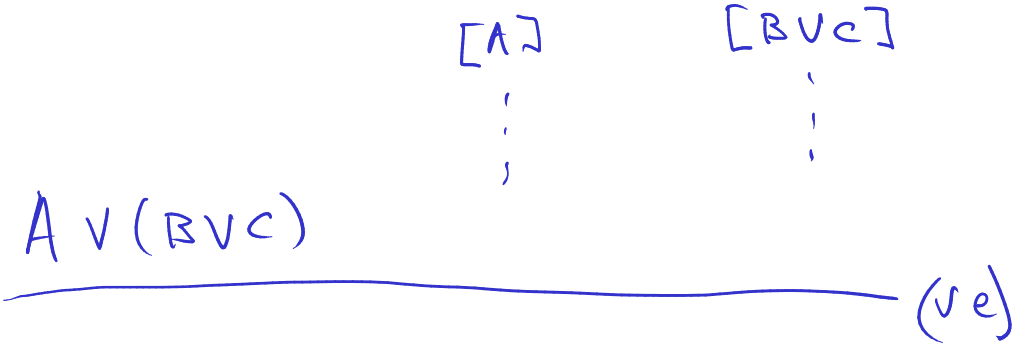
$$\frac{B}{A \to B}\ (\to i)\,u$$

$$\left\{\begin{array}{l}\dfrac{\dfrac{\dfrac{}{A \vdash A}(Ax) \qquad \dfrac{}{B \vdash B}(Ax)}{B, A \vdash A \wedge B}(\wedge i)}{B, A \vdash B}(\wedge e)\end{array}\right\}$$

$$\frac{B, A \vdash B}{B \vdash A \to B}\ (\to i)$$

$$\frac{\Gamma_1 \vdash A \qquad \Gamma_2 \vdash B}{\Gamma_1, \Gamma_2 \vdash A \wedge B}\ (\wedge i)$$

$$\frac{\Gamma \vdash A_{i \in \{1,2\}}}{\Gamma \vdash A_1 \vee A_2}\ (\vee i)$$



$A \vee (B \vee C) \vdash (A \vee B) \vee C$

Exercício 18. *Sejam φ e ψ fórmulas da lógica proposicional. Construa uma prova para o sequente $\neg(\varphi \vee \psi) \vdash (\neg\varphi) \wedge (\neg\psi)$ na lógica proposicional minimal.*