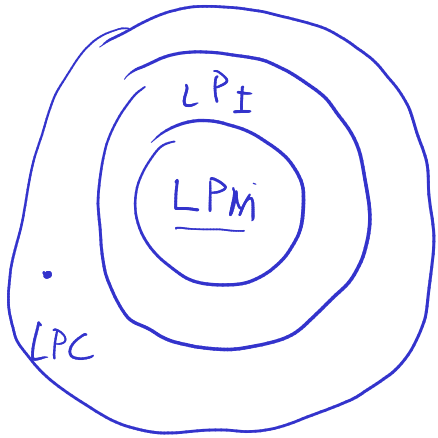


Exercício 45.  $\psi_1 \rightarrow \psi_2 \dashv\vdash_c (\neg\psi_1) \vee \psi_2$

①  $\psi_1 \rightarrow \psi_2 \vdash_c (\neg\psi_1) \vee \psi_2$



$$\begin{array}{c}
 \frac{\psi_1 \rightarrow \psi_2 \quad [\psi_1]^u}{(\neg\psi_2) \vee \psi_2} \quad (\rightarrow_e) \\
 \frac{[\neg\psi_2]^v \quad \psi_2}{\neg\psi_1} \quad (\neg_e) \\
 \frac{(\neg\psi_2) \vee \psi_2 \quad \frac{\perp}{\neg\psi_1} \quad (\neg_i)u}{(\neg\psi_1) \vee \psi_2} \quad (\vee_i) \\
 \frac{(\neg\psi_1) \vee \psi_2 \quad \frac{[\psi_2]^w}{(\neg\psi_1) \vee \psi_2} \quad (\vee_i)}{(\neg\psi_1) \vee \psi_2} \quad (\vee_e) \sigma, \omega
 \end{array}$$

②  $(\neg\psi_1) \vee \psi_2 \vdash_i \psi_1 \rightarrow \psi_2$

$$\begin{array}{c}
 \frac{[\neg\psi_1]^v \quad [\psi_1]^u}{\psi_2} \quad (\neg_e) \\
 \frac{(\neg\psi_1) \vee \psi_2 \quad \frac{\perp}{\psi_2} \quad (\perp_e)}{\psi_2} \quad (\vee_e) \sigma, \omega \\
 \frac{\psi_2}{\psi_1 \rightarrow \psi_2} \quad (\rightarrow_i)u
 \end{array}$$

$$\frac{[A]^u \dots B}{A \rightarrow B} (\rightarrow_i) u \qquad \frac{A \rightarrow B \quad A}{B} (\rightarrow_e)$$

falsch imp. der Lög. prop.

$$\frac{A \quad B}{A \wedge B} (\wedge_i)$$

$$\frac{A_1, \wedge A_2}{A_{i \in \{1,2\}}} (\wedge_e)$$

$$\frac{A_{i \in \{1,2\}}}{A_1 \vee A_2} (\vee_i)$$

$$\frac{A \vee B \quad [A]^u \quad [B]^v}{c} (\vee_e) u, v$$

$$\frac{[A]^u}{\perp} (\neg_i)$$

$$\frac{A \quad \neg A}{\perp} (\neg_e)$$

LPM

$$\frac{\perp}{A} (\perp_e)$$

LPI

$$\frac{[\neg A]^u}{\perp} (\text{PBC}) u$$

LPC.

$$\frac{}{A \vee (\neg A)} \text{LPI + PBC}$$

$$\frac{[\neg(A \vee \neg A)]^v \quad \frac{[A]^u}{A \vee \neg A} (\vee_i)}{A \vee \neg A} (\neg_e)$$

$$\frac{\perp}{\neg A} (\neg_i) u$$

$$\frac{[\neg(A \vee \neg A)]^v \quad \frac{\neg A}{A \vee \neg A} (\vee_i)}{A \vee \neg A} (\neg_e)$$

$$\frac{\perp}{A \vee \neg A} (\text{PBC}) v$$

**Exercício 22.** Sejam  $\varphi$  e  $\gamma$  fórmulas da lógica proposicional. Construa uma prova para o seqüente  $(\neg\varphi) \vee (\neg\gamma) \vdash \neg(\varphi \wedge \gamma)$  na lógica proposicional minimal.

