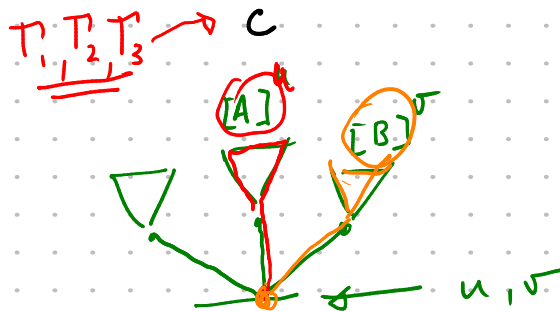
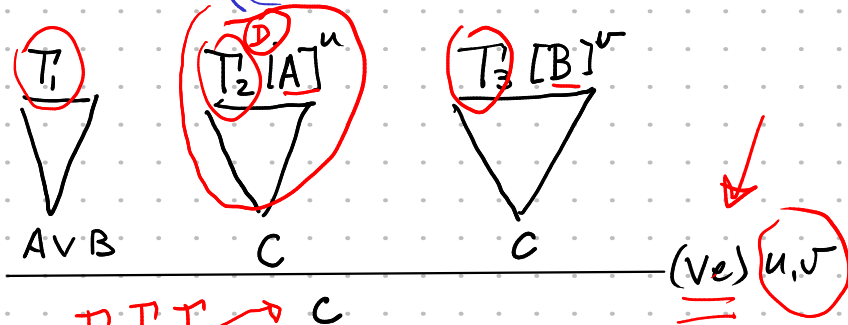
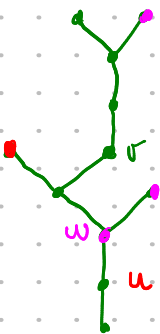


Exercício:  $\vdash_c ((A \rightarrow B) \rightarrow A) \rightarrow A$

$\varphi \vdash \varphi^{Ax}$

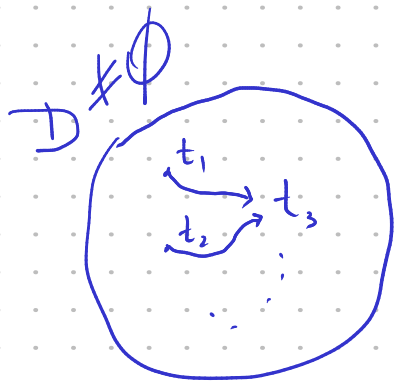
$(A \rightarrow B) \rightarrow A$   $[A]^u$   $[\neg A]^w$   
 $\vdash_c$   $\frac{[A]^u \quad [\neg A]^w}{\perp} (\perp e)$   
 $\frac{\perp}{B} (\perp e)$   $A, \neg A \rightarrow$   
 $\frac{B}{A \rightarrow B} (\rightarrow i) \sigma$   $A, \neg A$   
 $\frac{A \rightarrow B}{A} (\rightarrow e)$   $\neg A$   
 $\frac{A}{\perp} (\perp i) \omega$   $[\neg A]^w$   
 $\frac{\perp}{\neg \neg A} (\neg \neg e)$   
 $\frac{\neg \neg A}{A} (\rightarrow i) \mu$   $(A \rightarrow B) \rightarrow A$   
 $\frac{A}{((A \rightarrow B) \rightarrow A) \rightarrow A} (\rightarrow i) \mu$   $[\varphi]^u$   
 $\frac{((A \rightarrow B) \rightarrow A) \rightarrow A}{\perp} (\perp i) \mu$   $\frac{\perp}{\neg \varphi} (\perp i) \mu$

$[A]^u \quad [\neg A]^w$   
 $\frac{[A]^u \quad [\neg A]^w}{\perp} (\perp e)$   
 $\frac{\perp}{B} (\perp e)$   
 $\frac{B}{A \rightarrow B} (\rightarrow i) \sigma$   
 $\frac{A \rightarrow B}{A} (\rightarrow e)$   
 $\frac{A}{\perp} (\perp i) \omega$   $[\neg A]^w$   
 $\frac{\perp}{A} (PBC) \omega$   
 $\frac{A}{((A \rightarrow B) \rightarrow A) \rightarrow A} (\rightarrow i) \mu$   $(\neg \varphi) \rightarrow \perp$   
 $\neg(\neg \varphi)$   $\frac{\perp}{\varphi} (PBC) \mu$   
 $\frac{\neg \neg \varphi}{\varphi} (\neg \neg e)$   
 $\neg \varphi = \varphi \rightarrow \perp$



$\varphi ::= p \mid \perp \mid (\neg \varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi)$   
 fórmulas

Termos.  $t ::= x \mid f(t, \dots, t)$



$$f(x_1, x_2) = x_1 + x_2$$

Fórmulas

$$\boxed{T = \perp \rightarrow \perp}$$

fórmula atômica

$\varphi ::= \perp \mid (\neg \varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi) \mid \underline{p(t_1, \dots, t_n)}$

$\forall x \varphi$  |  $\exists x \varphi$

$$\frac{\forall x \varphi}{\varphi[x/t]} (\forall e)$$

$$\frac{\varphi(x_0)}{\forall x \varphi} (\forall i)$$

onde  $x_0$  é "qualquer", i.e.  
 a prova de  $\varphi(x_0)$  não depende de hipótese não descartada contendo  $x_0$ .

$$\frac{\forall x \varphi(x)}{\varphi(t)} (\forall e)$$

$$\frac{\varphi(x_0)}{\exists x \varphi} \quad (\exists i)$$

$$\frac{\exists x \varphi \quad \boxed{[\varphi(x_0)]^u}}{\exists x \varphi} \quad (\exists e) u$$

onde  $x_0$  é uma variável  
nova que não ocorre  
em  $\varphi$ .

$$\text{Ex: } \neg(\forall x p(x)) \vdash \exists x \neg p(x)$$

$$\frac{[\neg \exists x \neg p(x)]^u \quad \frac{[\neg p(x_0)]^v}{\exists x \neg p(x)} (\exists i)}{\neg \exists x \neg p(x)} (\neg e)$$

$$\frac{\perp}{p(x_0)} \quad (\text{PBC}) v$$

$$\frac{\frac{p(x_0)}{\forall x p(x)} (\forall i) \quad \neg(\forall x p(x))}{\forall x p(x)} (\neg e)$$

$$\frac{\perp}{\exists x \neg p(x)} \quad (\text{PBC}) u$$