

Atividade 3:

$$\vdash_i \neg\neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi$$

$$\frac{\frac{\frac{[\varphi]^\omega \quad [\neg\varphi]^\exists}{\perp} (\neg e) \quad \perp}{\psi} (\perp e) \quad \psi}{\varphi \rightarrow \psi} (\rightarrow i)^\omega}{\varphi \rightarrow \psi} (\rightarrow e)$$


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$$\frac{[\neg\varphi]^\exists \quad \varphi}{\perp} (\perp e)$$

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$$\frac{[\neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi]^\omega \quad \frac{[\varphi]^\omega}{(\varphi \rightarrow \psi) \rightarrow \varphi} (\rightarrow i)^\omega}{(\varphi \rightarrow \psi) \rightarrow \varphi} (\rightarrow e)$$


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$$\frac{\perp}{\psi} (\perp e)$$

$$\frac{\psi}{\varphi \rightarrow \psi} (\rightarrow i)^\omega$$

$$\frac{}{\neg\neg((\varphi \rightarrow \psi) \rightarrow \varphi)} (\neg i)^\omega$$

$$\frac{\frac{\frac{[\varphi]^\omega \quad [\neg\varphi]^\exists}{\perp} (\neg e) \quad \perp}{\psi} (\perp e) \quad \psi}{\varphi \rightarrow \psi} (\rightarrow i)^\omega}{\varphi \rightarrow \psi} (\rightarrow e)$$


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$$\frac{\varphi}{[\neg\varphi]^\exists} (\neg e)$$

$$\frac{[\neg\neg((\varphi \rightarrow \psi) \rightarrow \varphi)]^\omega \quad \perp}{\neg((\varphi \rightarrow \psi) \rightarrow \varphi)} (\neg i)^\omega$$


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$$\frac{\perp}{\neg\neg\varphi} (\neg i)^\exists$$

$$\frac{\neg\neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \neg\neg\varphi}{\neg\neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi} (\rightarrow i)^\omega$$


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$$\frac{}{\neg\neg((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi} (Ex 3 \dagger)$$

$(\neg\neg A) \rightarrow (\neg\neg B) \vdash \neg\neg(A \rightarrow B)$

exercício 37

Handwritten proof for  $(\neg\neg A) \rightarrow (\neg\neg B) \vdash \neg\neg(A \rightarrow B)$  on grid paper. The proof uses nested assumptions and logical rules:

- Assumption  $(\neg\neg A)$  (circled in orange) is used to derive  $A$  via  $(\neg e)$ .
- Assumption  $(\neg\neg B)$  (circled in orange) is used to derive  $\neg B$  via  $(\neg i)$ .
- Assumption  $(A \rightarrow B)$  (circled in orange) is used to derive  $B$  via  $(\rightarrow e)$ .
- Assumption  $(\neg(A \rightarrow B))$  (circled in orange) is used to derive  $A$  via  $(\rightarrow i)$  and  $\neg B$  via  $(\rightarrow e)$ .
- Assumption  $(\neg\neg(A \rightarrow B))$  (circled in orange) is used to derive  $\neg\neg(A \rightarrow B)$  via  $(\neg i)$ .

Handwritten proof for  $\neg\neg((\varphi \rightarrow \psi) \rightarrow \varphi) \vdash ((\varphi \rightarrow \psi) \rightarrow \varphi) \rightarrow \varphi$  on lined paper. The proof uses nested assumptions and logical rules:

- Assumption  $(\neg\neg((\varphi \rightarrow \psi) \rightarrow \varphi))$  (circled in red) is used to derive  $((\varphi \rightarrow \psi) \rightarrow \varphi)$  via  $(\neg e)$ .
- Assumption  $(\neg((\varphi \rightarrow \psi) \rightarrow \varphi))$  (circled in red) is used to derive  $\varphi$  via  $(\rightarrow i)$  and  $(\rightarrow e)$ .
- Assumption  $((\varphi \rightarrow \psi) \rightarrow \varphi)$  (circled in red) is used to derive  $\varphi$  via  $(\rightarrow e)$ .
- Assumption  $(\neg((\varphi \rightarrow \psi) \rightarrow \varphi))$  (circled in red) is used to derive  $\neg\neg((\varphi \rightarrow \psi) \rightarrow \varphi)$  via  $(\neg i)$ .