Exercício 40 : $(\neg\neg e)$ $\dfrac{\neg\neg\varphi}{\varphi}$

① $\quad \bot \vdash \varphi$

$$\dfrac{\dfrac{(Ax)}{\bot, \neg\varphi \vdash \bot}\ (\neg i)}{\dfrac{\bot \vdash \neg\neg\varphi}{\bot \vdash \varphi}\ (\neg\neg e)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\bot \qquad [\neg\varphi]^a}{\bot \wedge (\neg\varphi)}\ (\wedge i)}{\bot}\ (\wedge e)}{\neg\neg\varphi}\ (\neg i)\, a}{\varphi}\ (\neg\neg e)$$

$$\boxed{\boxed{\dfrac{}{T \vdash \varphi}\ (A_x)\ ,\ \text{se}\ \varphi \in T}}$$

$$\dfrac{}{\varphi \vdash \varphi}\ (A_x) \qquad\qquad \dfrac{T \vdash \varphi}{\varphi, T \vdash \varphi}\ (\omega)$$

② $\quad \vdash_{i+(\neg\neg e)} \varphi \vee \neg\varphi.$

$$\dfrac{\dfrac{\dfrac{[\neg(\varphi\vee\neg\varphi)]^b \quad \dfrac{[\varphi]^a}{\varphi\vee\neg\varphi}\ (\vee i)}{\dfrac{\bot}{\neg\varphi}\ (\neg i)\, a}\ (\neg e)}{[\neg(\varphi\vee\neg\varphi)]^b \quad \dfrac{\neg\varphi}{\varphi\vee\neg\varphi}\ (\vee i)}{\dfrac{\dfrac{\bot}{\neg\neg(\varphi\vee\neg\varphi)}\ (\neg i)\, b}{\varphi\vee\neg\varphi}\ (\neg\neg e)}\ (\neg e)}$$

③ $(\neg \varphi) \to \bot \quad \vdash_{i+\neg e} \varphi$

$$\frac{(\neg \varphi) \to \bot \qquad [\neg \varphi]^a}{\bot}(\to e)$$
$$\frac{\bot}{\neg \neg \varphi}(\neg i)\,a$$
$$\frac{}{\varphi}(\neg e)$$

Exercício 39:

Tab 2.5

$$\frac{\varphi \qquad \psi}{\varphi \wedge \psi}(\wedge i) \qquad\qquad \frac{\varphi_1 \wedge \varphi_2}{\varphi_{i \in \{1,2\}}}(\wedge e)$$

$$\frac{\varphi_{i \in \{1,2\}}}{\varphi_1 \vee \varphi_2}(\vee i) \qquad \frac{\varphi \vee \psi \qquad \overset{[\varphi]^a}{\underset{\chi}{\vdots}} \qquad \overset{[\psi]^b}{\underset{\chi}{\vdots}}}{\chi}(\vee e)\,a,b$$

$$\frac{\overset{[\varphi]^a}{\underset{\psi}{\vdots}}}{\varphi \to \psi}(\to i)\,a \qquad \frac{\varphi \to \psi \qquad \varphi}{\psi}(\to e)$$

$$\frac{\overset{[\varphi]^a}{\underset{\bot}{\vdots}}}{\neg \varphi}(\neg i)\,a \qquad \frac{\neg \varphi \qquad \varphi}{\bot}(\neg e)$$

$$\frac{\overset{[\neg \varphi]^a}{\underset{\bot}{\vdots}}}{\varphi}(PPc)\,a$$

(1) $\bot \vdash_{Int + PPC} \varphi$

$$\cfrac{\cfrac{\cfrac{\bot \qquad [\neg \varphi]^a}{\bot \land (\neg \varphi)}(\land i)}{\bot}(\land e)}{\varphi}(PPC)\, a$$

(2) $\vdash_{Int + PPC} \varphi \lor \neg \varphi$

$$\cfrac{\cfrac{\cfrac{\cfrac{[\neg(\varphi \lor \neg\varphi)]^b \qquad \cfrac{[\varphi]^a}{\varphi \lor \neg\varphi}(\lor i)}{\bot}(\neg e)}{\cfrac{\neg\varphi}{\varphi \lor \neg\varphi}(\lor i)}{(\neg i)\, a} \qquad [\neg(\varphi \lor \neg\varphi)]^b}{\bot}(\neg e)}{\varphi \lor \neg\varphi}(PPC)\, b$$

(38) $\vdash_i \neg\neg(((\varphi \to \psi) \to \psi) \to \varphi)$

$$\Gamma \quad \psi \vdash_i \varphi \to \psi \,\Big\}\; \cfrac{\psi}{\varphi \to \psi}(\to i)^\phi$$

$$\cfrac{\cfrac{\cfrac{\cfrac{[\varphi]^a \quad \psi}{\varphi \land \psi}(\land i)}{\psi}(\land e)}{\varphi \to \psi}(\to i)\, a}{} \qquad \bot$$

$$\dfrac{[\neg(((\varphi \to \psi) \to \varphi) \to \varphi)]^a \qquad \dfrac{\dfrac{[\varphi]^c}{((\varphi\to\psi)\to\varphi)\to\varphi}(\to_i)\phi}{\underset{}{}}(\neg e)}{}$$

$$[(\varphi\to\psi)\to\varphi]^b \qquad \dfrac{\dfrac{\dfrac{\bot}{\psi}(\bot e)}{\varphi\to\psi}(\to_i)c}{}(\to e)d$$

$$\dfrac{[\neg(((\varphi\to\psi)\to\varphi)\to\varphi)]^a \qquad \dfrac{\dfrac{\varphi}{((\varphi\to\psi)\to\varphi)\to\varphi}(\to_i)b}{}(\to e)}{\dfrac{\bot}{\neg\neg(((\varphi\to\psi)\to\varphi)\to\varphi)}(\neg i)a}(\neg e)$$

(37) $\vdash_i \neg\neg(\neg\neg\varphi \to \varphi)$

$$[\neg(\neg\neg\varphi\to\varphi)]^a \qquad \dfrac{[\varphi]^c}{\neg\neg\varphi\to\varphi}(\to_i)\phi \quad (\neg e)$$

$$[\neg\neg\varphi]^b \qquad \dfrac{\dfrac{\bot}{\neg\varphi}(\neg i)c}{}(\neg e)$$

$$\dfrac{\bot}{\varphi}(\bot e)$$

$$\dfrac{[\neg(\neg\neg\varphi\to\varphi)]^a \qquad \dfrac{\varphi}{\neg\neg\varphi\to\varphi}(\to_i)b}{\dfrac{\bot}{\neg\neg(\neg\neg\varphi\to\varphi)}(\neg i)a}(\neg e)$$

(36) $(\neg\neg\varphi) \to (\neg\neg\psi) \vdash_i \neg\neg(\varphi \to \psi)$

$$\frac{(\neg\neg\varphi)\to(\neg\neg\psi) \quad \dfrac{[\psi]^a}{\neg\neg\varphi}(\neg\neg i)}{\neg\neg\psi}(\to e)$$

$$\frac{[\neg(\varphi\to\psi)]^b \quad \dfrac{[\psi]^c}{\varphi\to\psi}(\to i)\,\phi}{\dfrac{\bot}{\neg\psi}(\neg i)\,c}(\neg e)$$

$$\dfrac{\bot}{\psi}(\bot e)$$

$$\dfrac{\varphi\to\psi}{}(\to i)\,a \qquad [\neg(\varphi\to\psi)]^b \qquad (\neg e)$$

$$\frac{A\leftrightarrow B}{A\to B}(\wedge e)$$

ou

$$\frac{A\leftrightarrow B}{B\to A}(\wedge e)$$

$$\dfrac{\bot}{\neg\neg(\varphi\to\psi)}(\neg i)\,b$$

(34) $\varphi\leftrightarrow\psi,\ \psi\to\chi,\ \neg\chi \vdash_{M} (\neg\varphi)\wedge(\neg\psi)$

$$\frac{[\varphi]^a \quad \dfrac{\varphi\leftrightarrow\psi}{\varphi\to\psi}(\wedge e)}{\dfrac{\psi}{}(\to e) \quad \psi\to\chi}(\to e)$$

$$\frac{\neg\chi \qquad \chi}{\dfrac{\bot}{\neg\varphi}(\neg i)\,a}(\neg e)$$

$$\frac{\psi\to\chi \quad \neg\chi}{\neg\psi}(MT)$$

$$(\neg\varphi)\wedge(\neg\psi)\ (\wedge i)$$

$$A \to B, \neg B \vdash \neg A$$

$$\frac{\displaystyle \frac{A \to B \quad [A]^x}{B} \; (\to e) \quad \neg B}{\displaystyle \frac{\bot}{\neg A} \; (\neg i)} \; (\neg e)$$