

Exercício 40 : $(\neg e) \frac{\neg \varphi}{\varphi}$

① $\perp \vdash \varphi$

$$\begin{array}{c}
 (\lambda_x) \quad \frac{}{\perp} \\
 \hline
 \frac{\perp, \neg \varphi \vdash \perp}{\perp \vdash \neg \varphi} (\lambda_e) \\
 \hline
 \frac{\perp \vdash \neg \varphi}{\perp \vdash \varphi} (\neg e)
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{\perp \quad [\neg \varphi]^a}{\perp \wedge (\neg \varphi)} (\lambda_i) \\
 \hline
 \frac{\perp \wedge (\neg \varphi)}{\perp} (\lambda_e) \\
 \hline
 \frac{\perp}{\neg \varphi} (\lambda_i) \alpha \\
 \hline
 \frac{\perp}{\varphi} (\neg e)
 \end{array}$$

$$\frac{}{\top \vdash \varphi} (\lambda_x), \text{ se } \varphi \in \Gamma$$

$$\frac{}{\varphi \vdash \varphi} (\lambda_x) \qquad \frac{\top \vdash \varphi}{\varphi, \top \vdash \varphi} (\omega)$$

② $\vdash_{i+(\neg e)} \varphi \vee \neg \varphi$.

$$\begin{array}{c}
 \frac{[\varphi]^a}{\neg(\varphi \vee \neg \varphi)} (\lambda_i) \\
 \hline
 \frac{\neg(\varphi \vee \neg \varphi) \quad \frac{[\varphi \vee \neg \varphi]^b}{\varphi \vee \neg \varphi} (\lambda_e)}{\perp} (\lambda_i) \alpha \\
 \hline
 \frac{\perp}{\neg \varphi} (\lambda_i) \beta \\
 \hline
 \frac{\neg \varphi \quad \frac{[\varphi \vee \neg \varphi]^b}{\varphi \vee \neg \varphi} (\lambda_e)}{\perp} (\lambda_i) \beta \\
 \hline
 \frac{\perp}{\neg(\varphi \vee \neg \varphi)} (\lambda_i) \beta \\
 \hline
 \frac{\neg(\varphi \vee \neg \varphi)}{\varphi \vee \neg \varphi} (\neg e)
 \end{array}$$

$$\textcircled{3} \quad (\neg\varphi) \rightarrow \perp \vdash_{i+ne} \varphi$$

$$\frac{(\neg\varphi) \rightarrow \perp \quad [\neg\varphi]^a}{\perp} (\rightarrow_e)$$

$$\frac{\perp}{\neg\neg\varphi} (\neg\neg_e)$$

$$\frac{\neg\neg\varphi}{\varphi} (\neg\neg\neg_e)$$

Exercício 39 :

Tab 2.5

$$\frac{\varphi \quad \varphi}{\varphi \wedge \varphi} (\wedge_i)$$

$$\frac{\varphi_1, 1 \varphi_2}{\varphi_i \in \{1, 2\}} (\wedge_e)$$

$$\frac{\varphi_{i \in \{1, 2\}} (\vee_i)}{\varphi_1 \vee \varphi_2}$$

$$\frac{\varphi \vee \varphi}{\begin{matrix} [\varphi]^a \\ \perp \end{matrix}} (\vee_e)$$

$$\frac{[\varphi]^a}{\varphi \rightarrow \varphi} (\rightarrow_e) a$$

$$\frac{\varphi \rightarrow \varphi \quad \varphi}{\varphi} (\rightarrow_e)$$

$$\frac{[\varphi]^a}{\perp} (\neg\neg_e) a$$

$$\frac{\neg\varphi}{\perp} (\neg_e)$$

$$[\neg\varphi]^a$$

$$\frac{\vdots}{\perp} (\text{PPC}) a$$

① $\perp \vdash \varphi$
 $\text{Int} + \text{PPC}$

$$\frac{\frac{\frac{\perp [\neg\varphi]^a}{\perp \wedge (\neg\varphi)}(\wedge_i)}{(\wedge_e)} \perp}{\varphi} (\text{PPC}) a$$

② $\vdash_{\text{Int} + \text{PPC}} \varphi \vee \neg\varphi$

$$\frac{\frac{\frac{[\neg(\varphi \vee \neg\varphi)]^b}{\varphi \vee \neg\varphi}(\vee_i)}{\perp (\neg_i) a} \neg\varphi}{\frac{\frac{\frac{\varphi \vee \neg\varphi}{[\neg(\varphi \vee \neg\varphi)]^b}(\neg_e)}{\perp} \perp}{\varphi \vee \neg\varphi} (\text{PPC}) b}$$

38 $\vdash_{\text{Int}} ((\varphi \rightarrow \varphi) \rightarrow \varphi) \rightarrow \varphi$

$$\Gamma \vdash \varphi \rightarrow \varphi \} \boxed{\frac{\varphi}{\varphi \rightarrow \varphi} (\rightarrow_i)}$$

$$\frac{[\varphi]^a \varphi}{\varphi \wedge \varphi} (\wedge_i) \frac{\varphi \wedge \varphi}{\varphi} (\wedge_e) \frac{\varphi}{\varphi \rightarrow \varphi} (\rightarrow_i) a$$

↓

$$\frac{\frac{\frac{[\neg((\varphi \rightarrow \varphi) \rightarrow \varphi) \rightarrow \varphi]^c}{(\varphi \rightarrow \varphi) \rightarrow \varphi} (\neg e)}{\frac{1}{\varphi} (\perp e)} c}{\frac{[\varphi \rightarrow \varphi]^b}{\varphi} (\rightarrow e) b} \rightarrow e$$

$$\frac{\frac{[\neg((\varphi \rightarrow \varphi) \rightarrow \varphi) \rightarrow \varphi]^c}{(\varphi \rightarrow \varphi) \rightarrow \varphi} (\neg e)}{\frac{1}{\varphi} (\perp e)} (\neg i) a$$

(37) $\vdash_i \neg\neg(\neg\neg\varphi \rightarrow \varphi)$

$$\frac{\frac{\frac{[\neg(\neg\neg\varphi \rightarrow \varphi)]^c}{\neg\neg\varphi \rightarrow \varphi} (\neg e) \phi}{\frac{1}{\neg\neg\varphi} (\perp e)} (\neg i) c}{\frac{[\neg\neg\varphi]^b}{\neg\neg\varphi} (\neg e)} (\neg i) b} \rightarrow e$$

$$\frac{\frac{[\neg(\neg\neg\varphi \rightarrow \varphi)]^c}{\neg\neg\varphi \rightarrow \varphi} (\neg e)}{\frac{1}{\varphi} (\perp e)} (\neg i) a$$

(36) $(\neg\varphi) \rightarrow (\neg\neg\varphi) \vdash_i \neg\neg(\varphi \rightarrow \varphi)$

$$\begin{array}{c}
 \frac{\frac{[\varphi]^a}{\neg(\varphi \rightarrow \psi)} b \quad \frac{[\varphi] c}{\varphi \rightarrow \psi} (\rightarrow) \phi}{\perp} (\neg i) c \\
 \frac{\frac{\perp}{\perp} (\neg i) c}{\perp} (\neg e) \\
 \frac{\frac{\frac{\perp}{\psi} (\perp e)}{\psi} (\rightarrow i) a}{\varphi \rightarrow \psi} (\rightarrow e) \\
 \frac{\frac{\varphi \rightarrow \psi}{[\neg(\varphi \rightarrow \psi)]^b}}{(\neg e)}
 \end{array}$$

$$\boxed{
 \begin{array}{c}
 A \leftrightarrow B \\
 \hline
 A \rightarrow B \quad (\rightarrow e) \\
 \text{on} \\
 \frac{A \leftrightarrow B}{B \rightarrow A} \quad (\rightarrow i)
 \end{array}
 }$$

$$\begin{array}{c}
 \perp \\
 \hline
 \neg(\varphi \rightarrow \psi) \quad (\neg i) b
 \end{array}$$

(34) $\varphi \leftrightarrow \psi, \psi \rightarrow \chi, \neg \chi \vdash_m (\neg \varphi) \wedge (\neg \psi)$

$$\begin{array}{c}
 \frac{[\varphi]^a \quad \frac{\frac{\varphi \leftrightarrow \psi}{\psi \rightarrow \chi} (\rightarrow e) \quad \frac{\psi \rightarrow \chi}{\perp} (\rightarrow e)}{\perp} (\neg i) a}{\perp} (\neg e) \\
 \frac{\perp}{\neg \varphi} \\
 \frac{\perp}{\neg \varphi} \quad \frac{\perp}{\neg \psi} (\neg i) a \\
 \hline
 \frac{\neg \varphi \quad \neg \psi}{(\neg \varphi) \wedge (\neg \psi)} (\wedge i)
 \end{array}$$

$A \rightarrow B, \neg B \vdash \neg A$

$$\frac{\frac{\frac{A \rightarrow B [A]^*}{B} (\neg e)}{\perp} (\neg i)}{\neg A}$$