

$$\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi$$

$$\begin{array}{c}
 \frac{\varphi \vee (\neg \varphi) \text{ (LEM)}}{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi} \quad \frac{[\varphi] \text{ (ass)}}{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi} \quad \frac{[\neg \varphi] \text{ (ass)}}{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi} \\
 \frac{\frac{\frac{\frac{[\varphi] \text{ (ass)}}{\vdash_c \perp} \text{ (}\perp\text{)}}{\vdash_c \varphi} \text{ (}\perp\text{e)}}{\vdash_c \varphi \rightarrow \neg \varphi} \text{ (}\rightarrow\text{i)}}{\vdash_c \varphi} \text{ (}\rightarrow\text{e)}}{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi)} \text{ (}\rightarrow\text{i)} \quad \frac{\frac{\frac{[\neg \varphi] \text{ (ass)}}{\vdash_c \perp} \text{ (}\perp\text{)}}{\vdash_c \varphi} \text{ (}\perp\text{e)}}{\vdash_c \varphi \rightarrow \neg \varphi} \text{ (}\rightarrow\text{i)}}{\vdash_c \varphi} \text{ (}\rightarrow\text{e)}}{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi)} \text{ (}\rightarrow\text{i)} \\
 \frac{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi \quad \vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi}{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi} \text{ (}\vee\text{e)} \text{ z, w}
 \end{array}$$

$$\left. \begin{array}{l}
 B \vdash_m A \rightarrow B \\
 \frac{[A]^u \quad B}{A \wedge B} \text{ (}\wedge\text{i)} \\
 \frac{A \wedge B}{B} \text{ (}\wedge\text{e)} \\
 \frac{B}{A \rightarrow B} \text{ (}\rightarrow\text{i)} u
 \end{array} \right\} \frac{B}{A \rightarrow B} \text{ (}\rightarrow\text{i)} \phi$$

$$\vdash_c \neg \neg ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi$$

$$\begin{array}{c}
 \frac{[\varphi] \text{ (ass)}}{\vdash_c \perp} \text{ (}\perp\text{)} \quad \frac{[\neg \varphi] \text{ (ass)}}{\vdash_c \perp} \text{ (}\perp\text{)} \\
 \frac{\vdash_c \perp}{\vdash_c \varphi} \text{ (}\perp\text{e)} \quad \frac{\vdash_c \perp}{\vdash_c \varphi} \text{ (}\perp\text{e)} \\
 \frac{\vdash_c \varphi}{\vdash_c \varphi \rightarrow \neg \varphi} \text{ (}\rightarrow\text{i)} \quad \frac{\vdash_c \varphi}{\vdash_c \varphi \rightarrow \neg \varphi} \text{ (}\rightarrow\text{i)} \\
 \frac{\vdash_c \varphi \rightarrow \neg \varphi}{\vdash_c \varphi} \text{ (}\rightarrow\text{e)} \quad \frac{\vdash_c \varphi \rightarrow \neg \varphi}{\vdash_c \varphi} \text{ (}\rightarrow\text{e)} \\
 \frac{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi)}{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi)} \text{ (}\rightarrow\text{i)} \quad \frac{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi)}{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi)} \text{ (}\rightarrow\text{i)} \\
 \frac{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi \quad \vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi}{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi} \text{ (}\vee\text{e)} \\
 \frac{\vdash_c ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi}{\vdash_c \neg \neg ((\varphi \rightarrow \neg \varphi) \rightarrow \varphi) \rightarrow \varphi} \text{ (}\neg\text{i)} u
 \end{array}$$