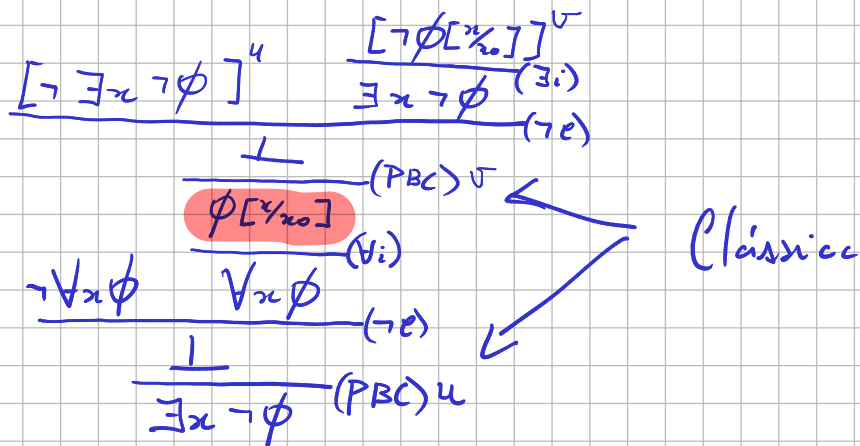


LC1 - 10/7/2024.

( $\neg\neg e$ ), (PBC), (LEM), (LP)

1. (a)  $\neg\forall x\phi \vdash \exists x\neg\phi$   
 $\neg\forall x\phi \vdash \exists x\neg\phi$



Lógica Clássica & Lógica Intuicionista.

Considere a seguinte afirmação:

Existem números irracionais  $x$  e  $y$  tais que  $x^y$  é racional.

Sabemos que  $\sqrt{2}$  é irracional.

Logo,  $\sqrt{2}^{\sqrt{2}}$  é racional ou irracional.

$\frac{A \vee \neg A}{LEM}$

Considere cada uma das possibilidades

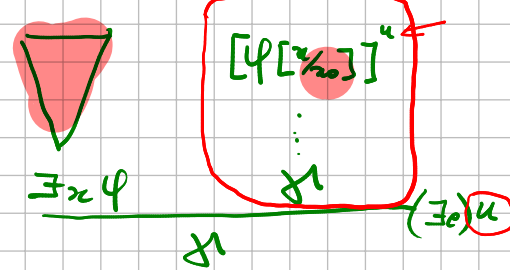
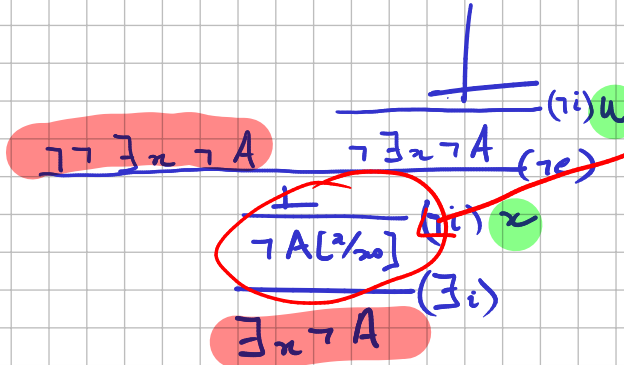
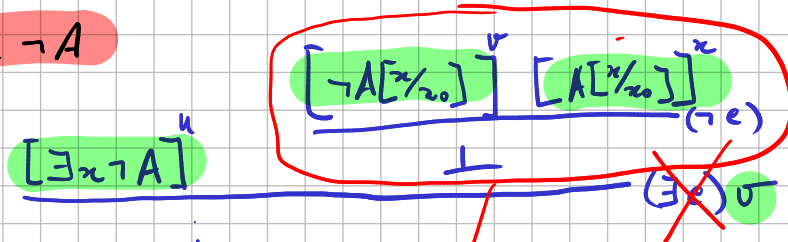
1.  $\sqrt{2}^{\sqrt{2}}$  é racional: Tome  $x = y = \sqrt{2}$

2.  $\sqrt{2}^{\sqrt{2}}$  é irracional:  $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^{\sqrt{2} \cdot \sqrt{2}} = (\sqrt{2})^2 = 2$ . Tome  $x = \sqrt{2}^{\sqrt{2}}$  e  $y = \sqrt{2}$ .

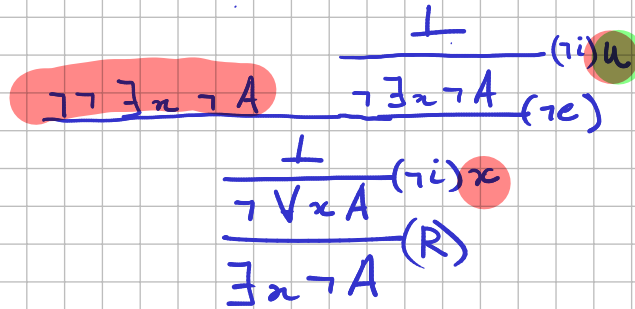
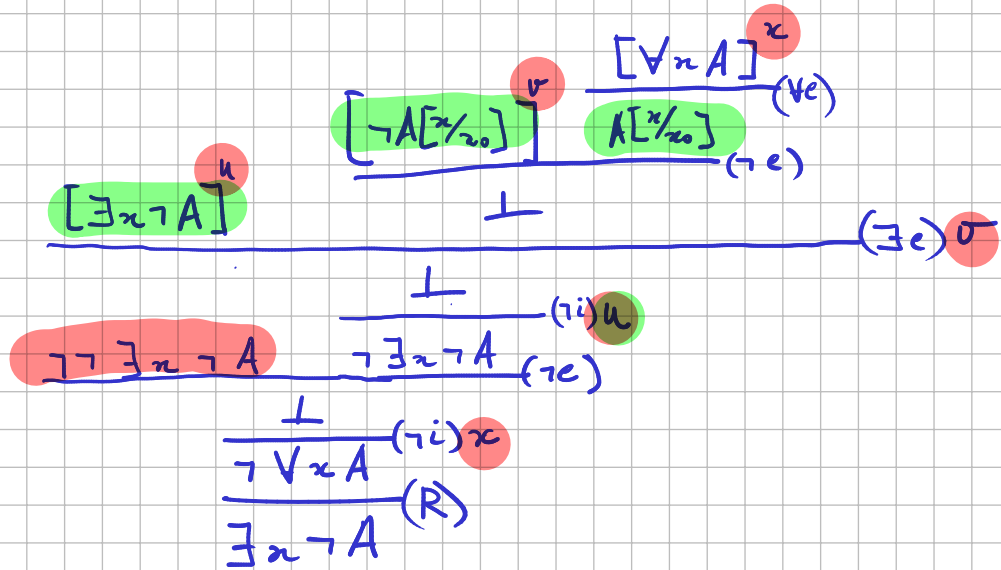
$\neg \forall x \phi \vdash \exists x \neg \phi$

$$\frac{\neg \forall x A}{\exists x \neg A} (R)$$

$\neg \neg \exists x \neg A \vdash \exists x \neg A$



onde  $x_0$  é variável nova que não ocorre em  $\phi$ .



$$\frac{\frac{\perp}{\neg \forall x A} (i)}{\exists x \neg A} (R)$$