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1  $x = A[r]$ ;
2  $i = p - 1$ ;
3 for  $j = p$  to  $r - 1$  do
4   if  $A[j] \leq x$  then
5      $i = i + 1$ ;
6     exchange  $A[i]$  com  $A[j]$ ;
7   end
8 end
9 exchange  $A[i + 1]$  with  $A[r]$ ;
10 return  $i + 1$ ;

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$$n = r - p + 1$$

Algorithm 9: $\text{partition}(A, p, r)$

$$T_p(n) = \sum_{j=1}^{n-1} 1 = n-1 = \Theta(n).$$

$$T_w(n) = T_w(n-1) + T_w(0) + \Theta(n)$$

$$= T_w(n-1) + \Theta(n)$$

$$T_w(n) = T_w(n-1) + c \cdot n \quad (c > 0).$$

$$= T_w(n-2) + c \cdot (n-1) + c \cdot n$$

$$= T_w(n-3) + c \cdot (n-2) + c \cdot (n-1) + c \cdot n$$

$$= \dots = T_w(n-n) + c \cdot 1 + c \cdot 2 + \dots + c \cdot (n-1) + c \cdot n$$

$$= c \cdot \sum_{i=1}^n i = c \cdot \frac{n \cdot (n+1)}{2} \Rightarrow \text{Af. } T_w(n) = \frac{c}{2} n(n+1)$$

Verificação por indução:

$$T_w(k+1) = T_w(k) + c \cdot (k+1).$$

$$\stackrel{\text{h.i.}}{=} \frac{c}{2} k(k+1) + c \cdot (k+1)$$

$$= \frac{c \cdot k(k+1) + c \cdot 2(k+1)}{2}$$

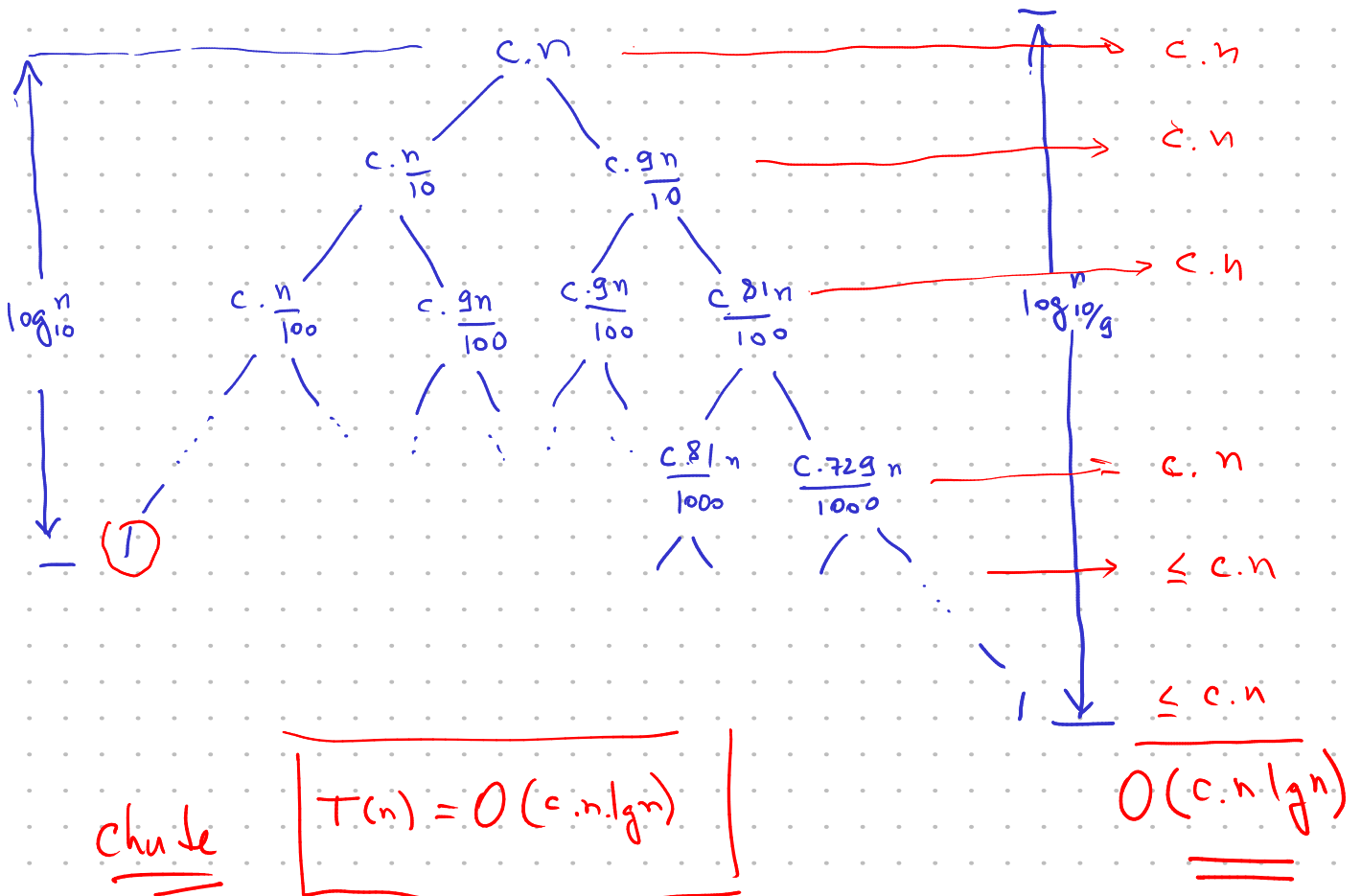
$$= \frac{c}{2} \cdot (k+1) \cdot (k+2)$$

$$\text{PO } \frac{\forall k (P_k \rightarrow P_{k+1})}{\forall n, P_n}$$

$$\Rightarrow T_w(n) = \Theta(n^2)$$



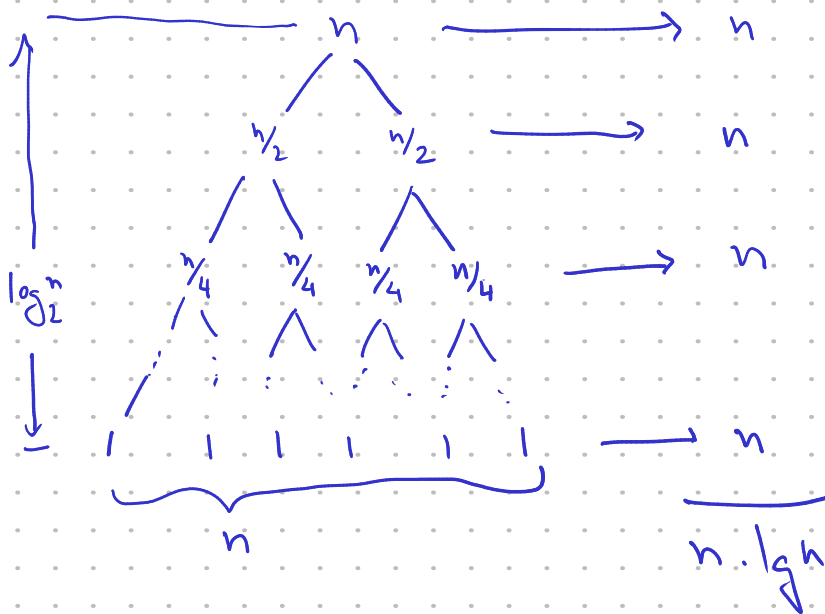
$$T(n) = T\left(\frac{9n}{10}\right) + T\left(\frac{n}{10}\right) + c \cdot n \quad (c > 0).$$



\Rightarrow Induktion p/ verifizieren.

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + n$$

$$n = 2^k$$



$$\Rightarrow T(n) = \Theta(n \cdot \lg n).$$