$T(1)=0$

Exercício 3. *Mostre que a recorrência* $T(1) = \Theta(1)$, $T(n) = 3T(n/3 + 5) + n/2$ *tem solução* $T(n) = O(n.\lg n)$.

$$\begin{cases} T(n) \leq c.n.\lg n - d & (c, d \text{ constantes}) \\ T(n) \leq c.(n-a).\lg(n-a) & (c, a \text{ constantes}) \end{cases}$$

Af: $\boxed{T(n) \leq c.(n-a).\lg(n-a)}$

Prova: Indução em n:

$$n > 1: \quad T(n) = 3.T(n/3 + 5) + n/2$$

$$\overset{h.i.}{\leq} 3.\left(c.\left(\frac{n}{3} + 5 - a\right).\lg\left(\frac{n}{3} + 5 - a\right)\right) + n/2$$

$$= c.(n + 15 - 3a).\lg\left(\frac{n + 15 - 3a}{3}\right) + n/2$$

$$= c.(n + 15 - 3a).\lg(n + 15 - 3a) \underbrace{- \lg 3 . c . (n + 15 - 3a) + n/2}_{\leq 0}$$

$\boxed{A + B \ (B \leq 0) \leq A}$

$\boxed{c.\lg 3 > 1/2}$

$$\leq c.(n + 15 - 3a).\lg(n + 15 - 3a)$$

$$\leq c.(n - a).\lg(n - a). \qquad (a \geq 15/2)$$

$\boxtimes$

$\in \mathbb{N}$

Af. $T(n) \le c \cdot n \cdot \lg n \quad (c > 0)$

Prova: Indução em n:

$n = 1: \quad T(1) \le 0. \quad \text{OK}.$

$$
\begin{aligned}
n > 1: \quad T(n) &= 3 \cdot T\left(n/3 + 5\right) + n/2 \qquad \left(n > n/3 + 5\right) \\
&\overset{h.i.}{\le} 3 \cdot \left(c \cdot \left(\frac{n}{3} + 5\right) \cdot \lg\left(n/3 + 5\right) \right) + n/2 \\
&= c \cdot (n + 15) \cdot \lg\left(\frac{n + 15}{3} \right) + n/2 \\
&= c\,(n + 15) \cdot \lg(n + 15) \underbrace{- c \cdot \lg 3 \cdot (n + 15) + n/2} \\
&\overset{?}{\le} c \cdot n \cdot \lg n
\end{aligned}
$$