

Exercício 79. Prove que $|\text{rev}(l)| = |l|$, qualquer que seja a lista l .

Exercício 80. Prove que $\text{rev}(l_1 \circ l_2) = (\text{rev}(l_2)) \circ (\text{rev}(l_1))$, quaisquer que sejam as listas l_1, l_2 .

Exercício 81. Prove que $\text{rev}(\text{rev}(l)) = l$, qualquer que seja a lista l .

Soluções 80: Indução em l_1 :

① $l_1 \triangleq \text{nil}$:

$$\begin{aligned} \text{rev}(l_1 \circ l_2) &\stackrel{*}{=} \text{rev}(\text{nil} \circ l_2) \stackrel{\text{def. } \circ}{=} \text{rev}(l_2) \stackrel{\substack{(\text{77}) \\ \text{ex. 2}}}{=} \text{rev}(l_2) \circ \text{nil} \stackrel{\text{def. rev}}{=} \\ &\text{rev}(l_2) \circ \text{rev}(\text{nil}) \stackrel{*}{=} \text{rev}(l_2) \circ \text{rev}(l_1). \end{aligned}$$

② $l_1 \triangleq h :: l'_1$:

$$\begin{aligned} \text{rev}(l_1 \circ l_2) &\stackrel{\Delta}{=} \text{rev}(\underbrace{h :: l'_1}_{\text{def. } \circ} \circ l_2) \stackrel{\text{def. } \circ}{=} \text{rev}(h :: \underbrace{l'_1 \circ l_2}_{\text{def. rev}}) \stackrel{\text{def. rev}}{=} \\ &\underbrace{(\text{rev}(l'_1 \circ l_2)) \circ (h :: \text{nil})}_{\text{h.i.}} \stackrel{\substack{(\text{76}) \\ \text{ex. 3}}}{=} (\text{rev}(l_2) \circ \text{rev}(l'_1)) \circ (h :: \text{nil}) \stackrel{\text{def. rev}}{=} \\ &\text{rev}(l_2) \circ \underbrace{(\text{rev}(l'_1) \circ (h :: \text{nil}))}_{\text{def. rev}} \stackrel{\Delta}{=} \text{rev}(l_2) \circ \text{rev}(h :: l'_1) \stackrel{\Delta}{=} \text{rev}(l_2) \circ \text{rev}(l_1). \end{aligned}$$

