Lema 26: $\lim\limits_{n\to\infty} \frac{f(n)}{g(n)} = c < \infty \implies$

$$f(n) = O(g(n)).$$

Prova: Temos $\lim\limits_{n\to\infty} \frac{f(n)}{g(n)} = c < \infty \iff$

$$\forall \varepsilon > 0,\ \exists n_0 > 0 : \left| \frac{f(n)}{g(n)} - c \right| < \varepsilon,\ \forall n > n_0 \iff$$

$$\underbrace{-\varepsilon < \frac{f(n)}{g(n)} - c < \varepsilon}, \quad \forall n > n_0$$

$$\frac{f(n)}{g(n)} < c + \varepsilon, \quad \forall n > n_0 \iff$$

$$f(n) < (c+\varepsilon) \cdot g(n), \quad \forall n > n_0$$

$$\Downarrow$$

$$f(n) \leq (c+\varepsilon) \cdot g(n), \quad \forall n \geq n_0 + 1$$

$$\iff$$

$$\boxed{f(n) = O(g(n))}$$

Ex: $n = O(n^2)$.

$$\lim_{n\to\infty} \frac{n}{n^2} = \lim_{n\to\infty} \frac{1}{n} = 0.$$

(91) $f(n) = O(g(n))$ e $g(n) = O(h(n)) \Rightarrow$

$f(n) = O(h(n))$.

$$f(n) = O(g(n)) \iff \exists \overset{\in\mathbb{R}}{c_1}, \overset{\in\mathbb{N}}{n_1} > 0 : f(n) \leq c_1 \cdot g(n), \forall n \geq n_1$$

$$g(n) = O(h(n)) \iff \exists \overset{\in\mathbb{R}}{c_2}, \overset{\in\mathbb{N}}{n_2} > 0 : g(n) \leq c_2 \cdot h(n), \forall n \geq n_2.$$

Queremos mostrar que $f(n) = O(h(n))$, ou seja, precisamos encontrar constantes positivas c e n_0 tais que:

$$\boxed{f(n) \leq c \cdot h(n), \ \forall n \geq n_0.}$$

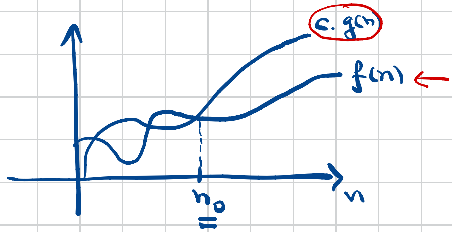
Temos $f(n) \leq c_1 \cdot g(n)$, $\forall n \geq \max(n_1, n_2)$

e $c_1 \cdot g(n) \leq c_1 \cdot c_2 \cdot h(n)$, $\forall n \geq \max(n_1, n_2)$

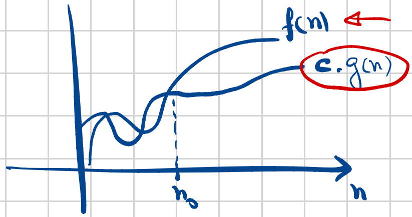
$$\Rightarrow f(n) \leq \underbrace{c_1 \cdot c_2}_{c} \cdot h(n), \ \forall n \geq \underbrace{\max(n_1, n_2)}_{n_0}$$

Tome

$$f(n) = O(g(n)) \iff \exists\, c, n_0 > 0 : f(n) \leq c \cdot g(n),\ \forall n \geq n_0$$

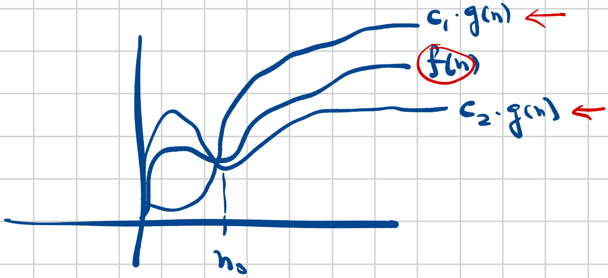


$$f(n) = \Omega(g(n)) \iff \exists\, c, n_0 > 0 : f(n) \geq c \cdot g(n),\ \forall n \geq n_0$$



$$f(n) = \Theta(g(n)) \iff \exists\, c_1, c_2, n_0 > 0 :$$

$$c_2 \cdot g(n) \leq f(n) \leq c_1 \cdot g(n),\ \forall n \geq n_0$$



$$T^{w}_{ins}(n) = \underline{O(n)}$$

$$\underline{is\ l} = \begin{cases} l, & l = nil \\ \underline{\underline{ins}}\ h\ (\underline{is\ l'}), & l = h :: l' \end{cases}$$

$$T^{w}_{is}(n) = T^{w}_{is}(n-1) + \underbrace{\underbrace{T^{w}_{ins}(n-1)}_{O(n-1)}}_{O(n)}$$

$$\boxed{T^{w}_{is}(n) = T^{w}_{is}(n-1) + O(n)}$$ eq. recorrência