

# Revisão da Lógica Proposicional (24/06/24).

→ Fragmento implicacional (Dedução Natural - G. Gentzen ~1930)

$$\Rightarrow \boxed{\varphi ::= p \mid \varphi \rightarrow \varphi}$$

$$\frac{H_1 \ H_2 \ \dots \ H_n \text{ (n>0)}}{C} (R)$$

← regra de inferência sequente.

## INTRODUÇÃO

$$\frac{[\varphi]^u \ \vdots \ \psi}{\varphi \rightarrow \psi} (\rightarrow_i)^u \checkmark$$

## ELIMINAÇÃO

$$\frac{\varphi \rightarrow \psi \quad \varphi}{\psi} (\rightarrow_e) \checkmark$$

$$H_1, H_2, \dots, H_n \vdash C$$

Exemplo:  $\vdash a \rightarrow a$

$$\frac{[a]^u \text{ (intro)}}{a \rightarrow a} (\rightarrow_i)^u \text{ (descente)}$$

Exemplo:  $a \rightarrow b, b \rightarrow c \vdash a \rightarrow c$

$$\frac{\frac{\frac{a \rightarrow b \quad [a]^u}{b} (\rightarrow_e) \quad b}{c} (\rightarrow_e)}{a \rightarrow c} (\rightarrow_i)^u$$

# Lógica Proposicional Minimal

$$\boxed{\neg \varphi \equiv \varphi \rightarrow \perp}$$

$$\varphi ::= p \mid \perp \mid \varphi \rightarrow \varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \underline{\underline{\neg \varphi}}$$

## INTRO

## ELIM

$$\left\{ \begin{array}{l} \frac{[\varphi]^u \ \vdots \ \psi}{\varphi \rightarrow \psi} (\rightarrow_i)^u \\ \frac{\varphi \quad \varphi}{\varphi \wedge \varphi} (\wedge_i) \end{array} \right.$$

$$\frac{\varphi \rightarrow \psi \quad \varphi}{\psi} (\rightarrow_e)$$

$$\frac{\varphi \wedge \psi}{\varphi} (\wedge_e) \quad \frac{\varphi \wedge \psi}{\psi} (\wedge_e)$$

$$\frac{\varphi_1 \wedge \varphi_2}{\varphi_{i_1 \wedge i_2}}$$

$$\varphi_{i_1 \wedge i_2}$$

$$\frac{\frac{\varphi \quad \varphi}{\varphi \vee \varphi} (\vee_i) \quad \frac{\varphi \quad \varphi}{\varphi \vee \varphi} (\vee_i)}{\frac{\varphi_{i_1 \vee i_2}}{\varphi_1 \vee \varphi_2} (\vee_i)}$$

$$\frac{\varphi \vee \psi \quad \begin{array}{c} [\varphi]^u \\ \vdots \\ \delta \end{array} \quad \begin{array}{c} [\psi]^v \\ \vdots \\ \delta \end{array}}{\delta} (\vee_e)^u, \sigma$$

$$\frac{[\psi]^u}{\perp} \frac{\perp}{\neg\psi} (\neg i) u$$

$$\frac{\neg\psi \quad \psi}{\perp} (\neg e)$$

Exemplo:  $\vdash a \rightarrow (b \rightarrow a)$

$$\frac{\frac{\frac{[a]^u \quad [b]^v}{a \wedge b} (\wedge e)}{a} (\rightarrow i) v}{b \rightarrow a} (\rightarrow i) u}{a \rightarrow (b \rightarrow a)} (\rightarrow i) u$$

Exemplo  $\psi \vdash \psi \rightarrow \psi$

$$\frac{\psi}{\psi \rightarrow \psi} (\rightarrow i) \psi$$

$$\frac{\frac{[\psi]^u \quad \psi}{\psi \wedge \psi} (\wedge e)}{\psi} (\rightarrow i) u$$

$$\frac{\psi}{\psi \rightarrow \psi} (\rightarrow i) u$$

$$\frac{}{\psi \vdash \psi} (Ax)$$

$$\frac{}{\psi, \top \vdash \psi} (Ax')$$

$$\frac{\frac{\frac{}{\psi \vdash \psi} (Ax)}{\psi, \psi \vdash \psi} (\wedge)}{\psi \vdash \psi \rightarrow \psi} (\rightarrow i)$$

Exemplo:  $(\neg\neg A) \wedge (\neg\neg B) \vdash \neg\neg(A \wedge B)$

$$\frac{\frac{\frac{(\neg\neg A) \wedge (\neg\neg B)}{\neg\neg A} (\wedge e)}{\neg\neg A} (\neg e) \quad \frac{\frac{(\neg\neg A) \wedge (\neg\neg B)}{\neg\neg B} (\wedge e)}{\neg\neg B} (\neg e)}{\frac{\perp}{\neg\neg(A \wedge B)} (\neg i) u} (\neg e)$$

# Lógica Proposicional Intuicionista

$$\text{LPM} + \boxed{\frac{\perp}{\varphi} (\perp e)}$$

Exemplo :

# Lógica Proposicional Clássica

$$\text{LPI} + \boxed{(\neg e) \frac{\neg\neg\varphi}{\varphi}}$$

Exemplo:  $(\neg\neg A) \wedge (\neg\neg B) \vdash \neg\neg(A \wedge B)$

$$\begin{array}{c} \frac{(\neg\neg A) \wedge (\neg\neg B)}{\neg\neg A} (\wedge e) \qquad \frac{(\neg\neg A) \wedge (\neg\neg B)}{\neg\neg B} (\wedge e) \\ \frac{(\neg\neg)}{\quad} \qquad \frac{\quad}{B} (\neg\neg e) \\ \frac{\quad}{A} \qquad \frac{\quad}{B} (\wedge i) \\ \frac{[\neg(A \wedge B)]^u \quad A \wedge B}{\quad} (\neg e) \\ \frac{\perp}{\neg\neg(A \wedge B)} (\neg i)^u \end{array}$$