

Revisão da Lógica Proposicional (24/06/24).

→ Fragmento implicacional (Dedução Natural - G. Gentzen ~1930)

$$\Rightarrow \boxed{\varphi ::= p \mid \varphi \rightarrow \varphi}$$

$$\frac{H_1 \ H_2 \ \dots \ H_n \text{ (n>0)}}{C} (R)$$

← regra de inferência sequente.

INTRODUÇÃO

$$\frac{[\varphi]^u \ \vdots \ \psi}{\varphi \rightarrow \psi} (\rightarrow_i)^u \checkmark$$

ELIMINAÇÃO

$$\frac{\varphi \rightarrow \psi \quad \varphi}{\psi} (\rightarrow_e) \checkmark$$

$$H_1, H_2, \dots, H_n \vdash C$$

Exemplo: $\vdash a \rightarrow a$

$$\frac{[a]^u \text{ intro}}{a \rightarrow a} (\rightarrow_i)^u \text{ desquite}$$

Exemplo: $a \rightarrow b, b \rightarrow c \vdash a \rightarrow c$

$$\frac{\frac{\frac{a \rightarrow b \quad [a]^u}{b} (\rightarrow_e) \quad b}{c} (\rightarrow_e)}{a \rightarrow c} (\rightarrow_i)^u$$

Lógica Proposicional Minimal

$$\boxed{\neg \varphi \equiv \varphi \rightarrow \perp}$$

$$\varphi ::= p \mid \perp \mid \underline{\varphi \rightarrow \varphi} \mid \underline{\varphi \wedge \varphi} \mid \underline{\varphi \vee \varphi} \mid \underline{\underline{\neg \varphi}}$$

INTRO

ELIM

$$\frac{[\varphi]^u \ \vdots \ \psi}{\varphi \rightarrow \psi} (\rightarrow_i)^u \checkmark$$

$$\frac{\varphi \rightarrow \psi \quad \varphi}{\psi} (\rightarrow_e) \checkmark$$

$$\frac{\varphi \quad \varphi}{\varphi \wedge \varphi} (\wedge_i) \checkmark$$

$$\frac{\varphi \wedge \psi}{\varphi} (\wedge_e) \quad \frac{\varphi \wedge \psi}{\psi} (\wedge_e)$$

$$\frac{\varphi_1 \wedge \varphi_2}{\varphi_{i \in \{1,2\}}} \checkmark$$

$$\frac{\frac{\varphi \quad \psi}{\varphi \vee \psi} (\vee_i) \quad \frac{\varphi \quad \psi}{\varphi \vee \psi} (\vee_i)}{\varphi_{i \in \{1,2\}} \vee \psi_{j \in \{1,2\}}} (\vee_i) \checkmark$$

$$\frac{\varphi \vee \psi \quad [\varphi]^u \ \vdots \ \chi \quad [\psi]^u \ \vdots \ \chi}{\chi} (\vee_e)^u \checkmark$$

$$\frac{\perp}{\neg \varphi} \text{ (}\neg\text{i)} \checkmark$$

$$\frac{\neg \varphi \quad \varphi}{\perp} \text{ (e)} \checkmark$$

Exemplo: $\vdash a \rightarrow (b \rightarrow a)$

$$\frac{\frac{\frac{[a]^k \quad [b]^r \text{ (}\neg\text{i)}}{a \wedge b \text{ (}\wedge\text{e)}}{a} \text{ (}\rightarrow\text{:)} \checkmark}{b \rightarrow a} \text{ (}\rightarrow\text{:)} \checkmark}{a \rightarrow (b \rightarrow a)} \text{ (}\rightarrow\text{:)} \checkmark$$

Exemplo $\varphi \vdash \varphi \rightarrow \varphi$

$$\frac{\varphi}{\varphi \rightarrow \varphi} \text{ (}\rightarrow\text{:)} \checkmark$$

$$\frac{\frac{\frac{[\varphi]^k \quad \varphi \text{ (}\neg\text{i)}}{\varphi \wedge \varphi \text{ (}\wedge\text{e)}}{\varphi} \text{ (}\rightarrow\text{:)} \checkmark}{\varphi \rightarrow \varphi} \text{ (}\rightarrow\text{:)} \checkmark$$

$$\frac{\varphi \vdash \varphi \text{ (Ax)}}{\varphi, \neg \vdash \varphi \text{ (Ax)'}}$$

$$\frac{\frac{\frac{\varphi \vdash \varphi \text{ (Ax)}}{\varphi, \varphi \vdash \varphi \text{ (}\wedge\text{)}}{\varphi \vdash \varphi \rightarrow \varphi} \text{ (}\rightarrow\text{:)} \checkmark$$

Exemplo: $(\neg\neg A) \wedge (\neg\neg B) \vdash \neg\neg(A \wedge B)$

$$\frac{\frac{\frac{(\neg\neg A) \wedge (\neg\neg B) \text{ (Ax)}}{\neg\neg A} \text{ (}\wedge\text{e)}}{\neg\neg B} \text{ (}\wedge\text{e)}}{\perp} \text{ (}\neg\text{i)} \checkmark$$

$$\frac{\frac{[A]^k \quad [B]^r \text{ (}\neg\text{i)}}{A \wedge B \text{ (}\wedge\text{e)}}{\perp} \text{ (}\neg\text{i)} \checkmark$$

$$\frac{\frac{\frac{(\neg\neg A) \wedge (\neg\neg B) \text{ (Ax)}}{\neg\neg B} \text{ (}\wedge\text{e)}}{\neg\neg A} \text{ (}\wedge\text{e)}}{\perp} \text{ (}\neg\text{i)} \checkmark$$

$$\frac{\perp}{\neg\neg(A \wedge B)} \text{ (}\neg\text{e)} \checkmark$$

Lógica Proposicional Intuicionista

LPM + $\boxed{\frac{\perp}{\varphi} (\perp e)}$ regra de eliminação do absurdo intuicionista ou regra da explosão.

Exemplo: $(\neg\neg A) \rightarrow (\neg\neg B) \vdash \neg\neg(A \rightarrow B)$

$\frac{\perp}{\neg\neg A} (\neg i) \checkmark$	$\frac{[\neg(A \rightarrow B)]^u}{\perp} (\perp e)$	$\frac{[\neg(A \rightarrow B)]^u}{\perp} (\perp e)$
$\frac{(\neg\neg A) \rightarrow (\neg\neg B)}{\neg\neg A} (\rightarrow e)$	$\frac{[\neg(A \rightarrow B)]^u}{A \rightarrow B} (\rightarrow i) \checkmark$	$\frac{[\neg(A \rightarrow B)]^u}{A \rightarrow B} (\rightarrow i) \checkmark$
$\frac{\perp}{\neg\neg B} (\neg i) \checkmark$	$\frac{[\neg(A \rightarrow B)]^u}{A \rightarrow B} (\rightarrow e)$	$\frac{[\neg(A \rightarrow B)]^u}{A \rightarrow B} (\rightarrow e)$
$\frac{(\neg\neg A) \rightarrow (\neg\neg B)}{\neg\neg B} (\rightarrow e)$	$\frac{\perp}{\neg\neg(A \rightarrow B)} (\neg i) \checkmark$	$\frac{\perp}{\neg\neg(A \rightarrow B)} (\neg i) \checkmark$

Lógica Proposicional Clássica

PBC, LEM, LP

$$\text{LPI} + \frac{(\neg e) \quad \neg\neg\phi}{\phi}$$

Exemplo: $(\neg\neg A) \wedge (\neg\neg B) \vdash \neg\neg(A \wedge B)$

$$\frac{\frac{(\neg\neg A) \wedge (\neg\neg B)}{\neg\neg A} (\wedge e) \quad \frac{(\neg\neg A) \wedge (\neg\neg B)}{\neg\neg B} (\wedge e)}{\frac{(\neg\neg) \quad \frac{A \quad B}{A \wedge B} (\wedge i)}{\neg\neg(A \wedge B)} (\neg e)} (\neg i) \mu$$

Exercício: $(\neg\psi) \rightarrow \phi \vdash (\neg\phi) \rightarrow \psi$

$$\frac{(\neg\neg) \quad (\neg\neg) \rightarrow \phi \quad \frac{[\neg\phi]^k \quad \perp}{\neg\psi} (\neg i)}{\neg\neg\psi} (\neg e) \quad \frac{\perp}{\psi} (\neg i) \mu$$

$$\frac{(\neg\psi) \rightarrow \phi \quad \frac{[\neg\phi]^k \quad \perp}{\neg\psi} (\neg i)}{\neg\neg\psi} (\neg e) \quad \frac{\perp}{\psi} (\neg i) \mu \quad \frac{(\neg\psi) \rightarrow \phi \quad \frac{[\neg\phi]^k \quad \perp}{\neg\psi} (\neg i)}{\neg\neg\psi} (\neg e) \quad \frac{\perp}{\psi} (\neg i) \mu}{(\neg\phi) \rightarrow \psi} (\rightarrow i) \mu$$

$$\frac{(\neg\psi) \rightarrow \phi}{(\neg\phi) \rightarrow \psi} (R)$$

$$\frac{\neg\neg A}{A} (R) \quad \frac{\perp}{\perp} (I)$$

$$\boxed{\neg\psi \equiv \psi \rightarrow \perp}$$

$$\frac{(\neg A) \rightarrow \perp}{(\neg\perp) \rightarrow A} (R) \quad \frac{\perp}{\perp} (I)$$

$$\frac{[\neg A]^x \quad \dots \quad B}{A \rightarrow B} (\rightarrow i) \mu$$

$$\frac{\frac{\neg\neg A}{(\neg A) \rightarrow \perp} (R) \quad \frac{[\perp]^k}{\perp \rightarrow \perp} (\rightarrow i) \mu}{\perp \rightarrow \perp} (\rightarrow e) \quad \frac{\perp \rightarrow \perp}{A} (A)}$$

$$\boxed{A \rightarrow B \vdash (\neg B) \rightarrow (\neg A)}$$

$$\frac{\frac{\psi \rightarrow \psi \quad [\psi]^x}{\psi} (\rightarrow e) \quad \frac{\perp}{\neg\psi} (\neg i) \mu}{\neg\psi} (\neg e) \quad \frac{\psi \rightarrow \psi \quad \neg\psi}{\neg\psi} (MT) \quad \frac{A \rightarrow B \quad [\neg B]^x}{\neg A} (MT) \quad \frac{\neg A}{(\neg B) \rightarrow (\neg A)} (\rightarrow i) \mu$$

$$\frac{A \rightarrow B \quad [\neg B]^x}{\neg A} (MT) \quad \frac{\neg A}{(\neg B) \rightarrow (\neg A)} (\rightarrow i) \mu$$

Exercício: $(\neg\phi) \rightarrow \psi \vdash_c (\neg\psi) \rightarrow \phi$

$$\frac{[\neg\psi]^x \quad (\neg\phi) \rightarrow \psi}{\neg\neg\phi} \text{ (MT)}$$

$$\frac{\neg\neg\phi}{\phi} \text{ (}\neg\neg\text{e)}$$

$$\frac{\phi}{(\neg\psi) \rightarrow \phi} \text{ (}\rightarrow\text{:)}^x$$

$\neg\neg A \vdash A$

$$\frac{(\neg A) \rightarrow \perp}{(\neg\perp) \rightarrow A} \text{ (R)}$$

$$\frac{\perp \rightarrow \perp}{\neg\perp} \text{ (}\rightarrow\text{e)}$$

A

Exercício: $(\neg A) \rightarrow (\neg B) \vdash B \rightarrow A$

$$\frac{(\neg\psi) \rightarrow (\neg\phi)}{\psi \rightarrow \phi} \text{ (R')}$$

$$\frac{[B] \quad (\neg\neg B)}{\neg\neg B} \text{ (}\neg\neg\text{i)}$$

$$\frac{\neg\neg B \quad (\neg A) \rightarrow (\neg B)}{\neg\neg A} \text{ (MT)}$$

$$\frac{\neg\neg A}{A} \text{ (}\neg\neg\text{e)}$$

$$\frac{A}{B \rightarrow A} \text{ (}\rightarrow\text{i)}$$

$$\frac{\neg\neg A \vdash A}{\vdash_c (\neg\neg A) \rightarrow A} \text{ (R')}$$

\vdash_c
 \vdash_m
 \vdash_e
 R'

$$\frac{[\neg A]^y \quad [\neg\neg A]^x}{\perp} \text{ (}\neg\text{e)}$$

$$\frac{\perp}{\neg\neg\neg A} \text{ (}\neg\text{i)}^x$$

$$\frac{\neg\neg\neg A}{\neg A \rightarrow \neg\neg\neg A} \text{ (}\rightarrow\text{i)}^y$$

$$\frac{\neg A \rightarrow \neg\neg\neg A}{(\neg\neg A) \rightarrow A} \text{ (R')}$$