

$$\text{insert } x \ l = \begin{cases} x :: \text{nil}, & \text{se } l = \text{nil} \\ x :: l, & \text{se } l = h :: tl \text{ e } x \leq h \\ h :: (\text{insert } x \ tl), & \text{se } l = h :: tl \text{ e } x > h. \end{cases}$$

$$\text{is } l = \begin{cases} l, & \text{se } l = \text{nil} \\ \text{insert } h (\text{is } tl), & \text{se } l = h :: tl \end{cases}$$

Fixpoint insertion_sort l :=

match l with

| nil => l

| h :: tl => insert h (insertion_sort tl)

end.

Fixpoint insert (x:nat) (l: list nat) :=

match l with

| nil => x :: nil

| h :: tl => if x <=? h then (x :: l)

else (h :: (insert x tl))

end.

$$\begin{aligned} \text{is } (3 :: 2 :: 1 :: \text{nil}) &= \text{insert } 3 (\text{is } (2 :: 1 :: \text{nil})) \\ &= \text{insert } 3 (\text{insert } 2 (\text{is } (1 :: \text{nil}))) \\ &= \text{insert } 3 (\text{insert } 2 (\text{insert } 1 (\text{is } \text{nil}))) \\ &= \text{insert } 3 (\text{insert } 2 (\text{insert } 1 \ \text{nil})) \\ &= \text{insert } 3 (\text{insert } 2 \ (1 :: \text{nil})) \\ &= \text{insert } 3 \ (1 :: (\text{insert } 2 \ \text{nil})) \end{aligned}$$

$$\text{insert } 3 \ (1 :: (2 :: \text{nil})) =$$

$$1 :: (\text{insert } 3 \ (2 :: \text{nil})) =$$

$$1 :: (2 :: (\text{insert } 3 \ \text{nil})) =$$

$$1 :: 2 :: 3 :: \text{nil} \quad \checkmark$$

$$1 :: (\text{insert } 3 \ (\text{insert } 2 \ \text{nil})) =$$

$$1 :: (\text{insert } 3 \ (2 :: \text{nil}))$$

