

1.

$$\boxed{\gamma \gamma A \vdash_m \gamma A}$$

$$\frac{\frac{\frac{[A]^x}{\gamma \gamma A} \quad \frac{[\gamma A]^x}{\gamma \gamma A} \quad \frac{\perp}{\gamma \gamma A} \quad (i) \quad \sigma}{\gamma \gamma A} \quad (e)}{\gamma A} \quad (i) \quad \sigma$$

$$\frac{[A]^x}{A \rightarrow A} \quad (i) \quad \sigma$$

2.

$$A \rightarrow B \vdash_c (\neg A) \vee B$$

$$\frac{\frac{\frac{A \rightarrow B \quad [A]^u}{B} (\rightarrow_e) \quad \frac{A \rightarrow B \quad [A]^u}{(\neg A) \vee B} (\vee_i)}{A \vee (\neg A)} (\text{LEM})}{(\neg A) \vee B} (\vee_e) \text{ u.s.}$$

$$\frac{A \rightarrow B}{(\neg A) \vee B} (R)$$

$$\frac{\frac{[A]^u}{A \rightarrow A} (\rightarrow_i) \text{ u} \quad \frac{A \rightarrow A}{(\neg A) \vee A} (R)}{(\neg A) \vee A}$$

$$3. (\neg B) \rightarrow (\neg A) \vdash A \rightarrow B$$

$$\begin{array}{c}
 \frac{(\neg B) \rightarrow (\neg A)}{\neg \neg B} (E) \\
 \frac{\neg \neg B}{B} (E) \\
 \frac{B}{A \rightarrow B} (\rightarrow I)
 \end{array}$$

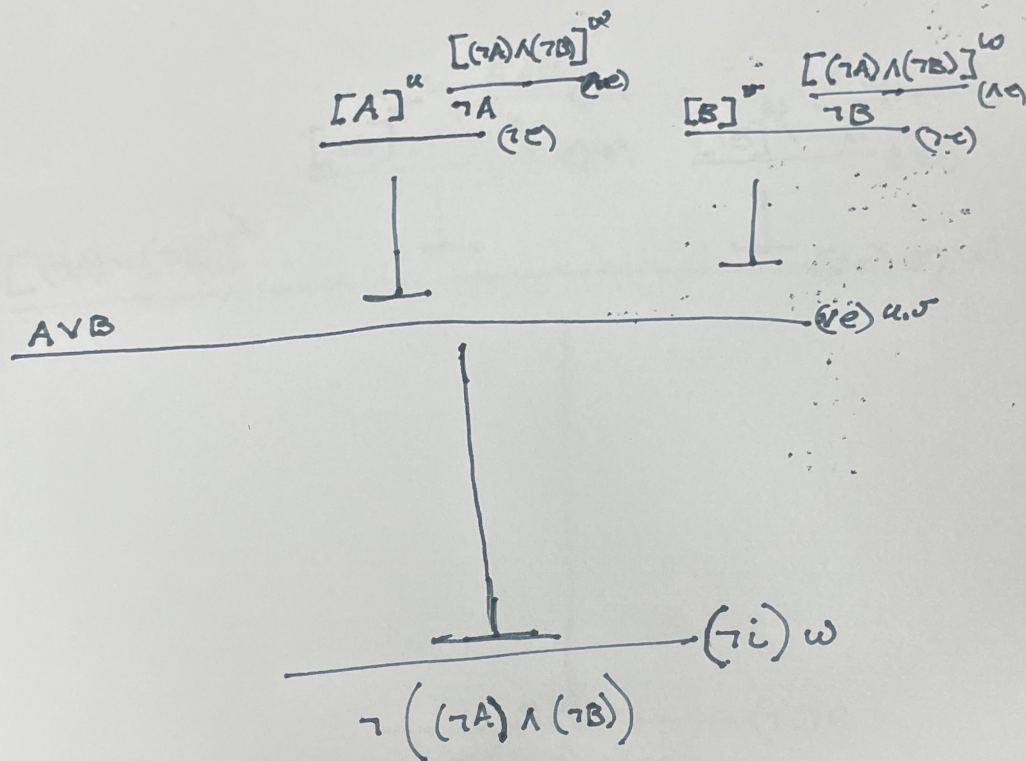
$$\begin{array}{c}
 \frac{(\neg B) \rightarrow (\neg A)}{A \rightarrow B} (R) \\
 \frac{A \rightarrow B}{\neg \neg A} (R) \\
 \frac{\neg \neg A}{A} (E)
 \end{array}$$

$$\neg \neg A \vdash A$$

Logo R é clássica.

4.

$$A \vee B \vdash \neg(\neg A \wedge \neg B)$$



5.

$$A \wedge B \vdash_{\text{M}} \neg((\neg A) \vee (\neg B))$$

$$\begin{array}{c}
 \begin{array}{cc}
 \frac{A \wedge B \quad (\wedge e)}{A \quad (\wedge e)} & \frac{A \wedge B \quad (\wedge e)}{B \quad (\wedge e)} \\
 \frac{A \quad (\wedge e)}{[\neg A]^\times} & \frac{B \quad (\wedge e)}{[\neg B]^\omega}
 \end{array} \\
 \frac{[\neg A]^\times \quad \perp \quad [\neg B]^\omega \quad \perp \quad (\vee e) \vee, \omega}{\neg((\neg A) \vee (\neg B))^\times} \\
 \frac{\neg((\neg A) \vee (\neg B))^\times}{\neg((\neg A) \vee (\neg B)) \quad (\neg i) \neg}
 \end{array}$$