

6. $A \vdash \neg \neg A$

$$\frac{\frac{(Ax) \quad \neg A \vdash \neg A \quad A \vdash A \quad (Ax)}{A, \neg A \vdash \perp} (\neg i)}{A \vdash \neg(\neg A)} (\neg e)$$

$$\frac{\neg \neg A \vdash A}{A \vdash \neg \neg A} (\neg \neg i)$$

$\neg \neg A \vdash A$

$$\frac{\varphi \vdash \varphi}{\Gamma, \varphi \vdash \varphi} (Ax)$$

$$\frac{\Gamma, \varphi \vdash \varphi}{\Gamma \vdash \varphi \rightarrow \varphi} (\rightarrow i)$$

$$\frac{\Gamma_1 \vdash \varphi \rightarrow \varphi \quad \Gamma_2 \vdash \varphi}{\Gamma_1 \cup \Gamma_2 \vdash \varphi} (\rightarrow e)$$

$$\frac{\Gamma, \varphi \vdash \perp}{\Gamma \vdash \neg \varphi} (\neg i)$$

$$\frac{\Gamma_1 \vdash \neg \varphi \quad \Gamma_2 \vdash \varphi}{\Gamma_1 \cup \Gamma_2 \vdash \perp} (\neg e)$$

$$\frac{(Ax) \quad \neg \neg A \vdash \neg \neg A \quad A \vdash A \quad (Ax)}{A \vdash \neg \neg A} (\neg \neg i)$$

$$\frac{A \vdash \neg \neg A \quad \neg \neg A \vdash A}{A \vdash A} (\neg \neg e)$$

(7) $A \rightarrow B \vdash (\neg \neg A) \rightarrow (\neg \neg B)$

$$\frac{(Ax) \quad A \rightarrow B \vdash A \rightarrow B \quad \neg B \vdash \neg B \quad (Ax)}{A \rightarrow B, \neg B \vdash \neg A} (MT)$$

$$\frac{A \rightarrow B, \neg B \vdash \neg A}{A \rightarrow B, \neg \neg A, \neg B \vdash \perp} (\neg e)$$

$$\frac{A \rightarrow B, \neg \neg A, \neg B \vdash \perp}{A \rightarrow B, \neg \neg A \vdash \neg \neg B} (\neg i)$$

$$\frac{A \rightarrow B, \neg \neg A \vdash \neg \neg B}{A \rightarrow B \vdash (\neg \neg A) \rightarrow (\neg \neg B)} (\rightarrow i)$$

$$\frac{\varphi \vdash \varphi}{\Gamma, \varphi \vdash \varphi} (Ax)$$

$$\frac{\Gamma, \varphi \vdash \varphi}{\Gamma \vdash \varphi \rightarrow \varphi} (\rightarrow i)$$

$$\frac{\Gamma_1 \vdash \varphi \rightarrow \varphi \quad \Gamma_2 \vdash \varphi}{\Gamma_1 \cup \Gamma_2 \vdash \varphi} (\rightarrow e)$$

$$\frac{\Gamma, \varphi \vdash \perp}{\Gamma \vdash \neg \varphi} (\neg i)$$

$$\frac{\Gamma_1 \vdash \neg \varphi \quad \Gamma_2 \vdash \varphi}{\Gamma_1 \cup \Gamma_2 \vdash \perp} (\neg e)$$

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$$\neg\neg(A \rightarrow B) \vdash (\neg\neg A) \rightarrow (\neg\neg B)$$

$$\begin{array}{c} \neg\neg(A \rightarrow B), \neg\neg A \vdash B \quad (\rightarrow_i) \\ \hline \neg\neg(A \rightarrow B) \vdash (\neg\neg A) \rightarrow B \quad (\rightarrow_e) \\ \hline \neg\neg(A \rightarrow B), \neg\neg A \vdash B \quad (\neg_i) \\ \hline \neg\neg(A \rightarrow B), \neg\neg A \vdash \neg\neg B \quad (\neg_i) \\ \hline \neg\neg(A \rightarrow B) \vdash (\neg\neg A) \rightarrow (\neg\neg B) \quad (\rightarrow_i) \end{array}$$

$$\begin{array}{c} \frac{\varphi \vdash \varphi}{\varphi \vdash \varphi} (Ax) \\ \frac{T, \varphi \vdash \varphi}{T \vdash \varphi \rightarrow \varphi} (\rightarrow_i) \\ \frac{T, \varphi \rightarrow \varphi \quad T_2 \vdash \varphi}{T, T_2 \vdash \varphi} (\rightarrow_e) \\ \frac{T, \varphi \vdash \perp}{T \vdash \neg \varphi} (\neg_i) \\ \frac{T \vdash \neg \varphi \quad T_2 \vdash \varphi}{T, T_2 \vdash \perp} (\neg_e) \end{array}$$

$$\frac{T \vdash \varphi}{T \vdash \neg\neg \varphi} (\neg_i)$$

$$\frac{T \vdash \varphi \rightarrow \varphi \quad T_2 \vdash \neg \varphi}{T, T_2 \vdash \neg \varphi} (MT)$$

$$\begin{array}{c} (Ax) \quad \frac{A \rightarrow B \vdash A \rightarrow B}{A \rightarrow B \vdash A} (MT) \\ \hline \neg\neg A \vdash \neg\neg A \quad (Ax) \\ \hline \neg\neg A, B, A \rightarrow B \vdash \perp \quad (\neg_i) \\ \hline \neg\neg A, B \vdash \neg(A \rightarrow B) \quad (\neg_e) \end{array}$$

$$(Ax) \quad \frac{\neg\neg(A \rightarrow B) \vdash \neg\neg(A \rightarrow B)}{\neg\neg(A \rightarrow B) \vdash \neg\neg(A \rightarrow B)} (Ax)$$

$$\frac{\neg\neg(A \rightarrow B), \neg\neg A, \neg B \vdash \perp}{\neg\neg(A \rightarrow B), \neg\neg A \vdash \neg\neg B} (\neg_i)$$

$$\frac{\neg\neg(A \rightarrow B), \neg\neg A \vdash \neg\neg B}{\neg\neg(A \rightarrow B) \vdash (\neg\neg A) \rightarrow (\neg\neg B)} (\rightarrow_i)$$